OPTIMAL AND SUBOPTIMAL CONTROL OF MULTIAREA LOAD FREQUENCY CONTROL SYSTEMS

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BY

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CERTIFICATE

Certified that this work, "Optimal and Suboptimal Control of Multiarea Load Frequency Control Systems" by Mr. V. Ramachandra Moorthi has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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SYNOPSIS

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Some computational methods to determine suboptimal controllers for multiarea Load Frequency Control Systems are presented and their nearness to the optimal controller is evaluated. Further some of the realistic factors which are usually assumed to be absent, for the sake of simplicity, are considered here in their complete shape and their effect on the response of the Load Frequency Control (LFC) system studied.

The small perturbation (linear) model of a multi-area LFC system is considered in Chapters 2 and 3 wherein two methods of reduction in computation are given. In the first method, a three-equal-area system comprising of 11 state variables is approximated as a two-unequal-area

system having 7 state variables. This is achieved by combining two of the areas (in the three-area system) into a single large area and treating it as connected to the third-area by an equivalent tieline. The optimal response of the reduced system is then determined and compared with that of the original system. The method is then extended to a three-unequal-area system which is approximated as a two-unequal-area system.

A second method of reduction in computation of optimal control of linear systems invokes the theory of aggregation of Aoki. In this method the original two-area-system comprising of 9 state variables is reduced to a system consisting of 6 state variables which has, as its eigenvalues, the dominant eigenvalues of the original system. The optimal feedback controller is then computed for the reduced system and is used as a suboptimal controller for the original system. The nearness of this suboptimal control to the optimal one is examined by comparing the following. a) The suboptimal responses with optimal ones, b) the suboptimal performance index with that of the optimal one and c) the stability of the closed-loop systems in both the cases. This method involves correct knowledge of the eigen-vectors of the original system.

In a multiarea LFC system, the tieline power flow between any two areas is a function of the sine of the angular difference between the voltage vectors at either

end of the tieline. For small perturbations, this angular difference is small and the tieline power flow can be taken as a function of the angular difference itself which makes the system linear. However, for large perturbations, the above assumption cannot be made and the system must be treated as nonlinear. In Chapters 4 and 5, advantage is taken of the trigonometrical sine term occuring as the nonlinear term in LFC systems and the method of reduction by aggregation is extended to reduce a higher order nonlinear system into a lower order one. The method of Lukes, given for regulation of autonomous nonlinear dynamical systems is then conveniently applied for the reduced nonlinear system. As in the case of linear systems, the optimal feedback controller of the reduced nonlinear system is utilized as a suboptimal controller for the original nonlinear system. With the help of a 4th order example, it is shown that the suboptimal response closely resembles the optimal one.

One of the usual assumptions that is made while computing optimal control of LFC systems is that there is no interaction between the Megavar-voltage and Megawatt-frequency control loops. This is not strictly true in the actual state of affairs. The effect of inclusion of excitation control on the frequency variation, which is obtained as a function of time, is studied in the last part of this thesis. A second realistic factor included in LFC systems and studied in this part is the construction of an observer in the event of nonmeasurability of some of the

state variables and its effect on the response of the state variables. It is shown that the error in the response thus obtained is small and also decays exponentially with time, thus achieving a near-optimal controller with the help of the observer. The above two aspects are covered in Chapters 6 and 7 of the thesis.

In the author's opinion, the following contributions are made in this thesis:

i) Elegant and efficient computational techniques are developed for the optimal and suboptimal control of linear multiarea LFC systems.

ii) Suboptimal regulation of a nonlinear multiarea LFC system is achieved by reducing the order of the nonlinear system to a computationally convenient size.

iii) The effect of excitation control on the response of the LFC system is determined taking into account, the interaction of the area voltage on the area load also.

iv) An observer is constructed when some of the state variables are assumed to be nonmeasurable and near-optimal control achieved by using the estimates of the state variables so obtained, in the optimal controller structure.

Finally, concluding remarks are made on all the computational methods presented in the thesis.

CHAPTER 1

INTRODUCTION

The study of Load Frequency Control (LFC) of multiarea systems is necessitated by the importance of maintenance of constant frequency and tieline power flows in large power networks. A disturbance in any part of the network has its effect on the frequency of the entire network. This problem has been studied by considering a "coherent" group of generators that swing together under disturbance, into a single control area. It is necessary to consider, as many areas as the number of such "coherent" groups. These areas are interconnected together by means of several tielines.

An LFC system has been treated in the past[1,2,9] as a conventionally controlled closed loop system with speed governor, turbine and equivalent generator represented by their transfer functions. The system is controlled by a signal which depends upon the frequency deviation, tieline power deviation and/or their integrals.

The advent of optimal control techniques has had considerable impact on the analysis of LFC systems also,

as is the case with many other control systems. By these techniques, the closed loop controller is computed in an optimum fashion as a linear combination of all the available state variables viz., frequency deviation and its integral, tieline power deviation and its integral, governor valve position and the turbine power output[3]. In this thesis, these techniques are extended to devise some elegant and efficient computational methods for the determination of suboptimal controllers for LFC systems.

1.2 BRIEF REVIEW OF PREVIOUS WORK DONE ON LOAD FREQUENCY CONTROL

The small perturbation model of a two-area LFC system was first studied on the analogue computer by Concordia and Kirchmayer[1,2]. The transfer functions of the governor, steam turbine and equivalent generator were simulated on the computer, and by taking a step load disturbance in one of the areas, the frequency deviation and tieline deviation were obtained as functions of time. Later Van Ness[4,22] studied the model of a large system consisting of eight hydroelectric generating stations, each having integral control. He also presented a method for finding the eigenvalues and their sensitivities to the parameters of the system. However, the above work considered the LFC system to be linear. For the first time Aggarwal and Bergseth[5] considered a nonlinear multiarea LFC system with conventional controller

under condition of large perturbation of load and applied Nonlinear Programming techniques to optimize the performance of the system.

Elgerd and Fosha[3] seem to have applied optimal control techniques for the first time to the linearized model of a two-area LFC system. The state variables in their study consisted of the frequency deviation and its integral, tieline/deviation and its integral, governor valve position and the turbine power output. They assumed that all the state variables are accessible for measurement, there is no noise in the system, and that there is no interaction between Megavar voltage and Megawatt frequency control loops.

It is the aim of the present thesis not only to seek methods of reduction in the computation of optimum controllers for multiarea LFC systems but also to study the LFC system without the simplifying assumptions made by Elgerd and Fosha[3].

1.3 OUTLINE OF VARIOUS CHAPTERS

In Chapter 2 a method is presented for the approximation of a three-equal-area system to a two-unequal-area system. The responses of the state variables of interest are determined in both cases and compared. Next, the approximation of a three-unequal-area system is taken up and studied in a similar manner.

In Chapter 3 a method for reduction in computation of multiarea LFC systems is presented. In it, the theory of aggregation due to Aoki[6] is invoked and the original system is reduced to one, of lesser dimensionality. In the process of reduction, it is ensured that the reduced system matrix has, as its eigenvalues, the dominant eigenvalues of the original system matrix. The optimum controller is then computed for the reduced system and is used as a suboptimal controller for the original system. The performance index and closed loop stability with suboptimal control are compared with those obtained by using the optimum controller.

One of the contributions made in this thesis is the suboptimal regulation of the large perturbation model of a two-area LFC system. This aspect is dealt with in Chapters 4 and 5. A multiarea LFC system under large load perturbations is essentially a nonlinear system in which the tieline power constitutes the non-linearity. Lukes[7] has presented a method for the regulation of autonomous nonlinear dynamical systems; but it is not convenient to apply the same to systems of, say 7th order and above. Here, by extending the theory of aggregation due to Aoki[6] to nonlinear LFC systems also, the 8th order nonlinear two-area LFC system is reduced to a 5th order nonlinear model. A suboptimal closed loop controller is then designed for the original

system by applying the method due to Lukes to the reduced system.

The effect of excitation control as well as the effect of load variation with voltage, on the dynamics of an LFC system are studied in Chapter 6. In Chapter 7 an observer of the Luenberger type[8] is constructed assuming that some of the state variables are nonmeasurable and the responses so obtained are compared with those obtained by Elgerd and Fosha[3].

Finally, in Chapter 8, concluding remarks are given on the different computational methods presented in the thesis; also, suggestions are given for further research in this inexhaustible area.

CHAPTER 2

REDUCTION OF LOAD FREQUENCY CONTROL SYSTEMS BY APPROXIMATION

The present day power systems are quite large in extent and for the purpose of load frequency control, they have to be grouped into a number of control areas depending on their electrical characteristics. For the application of optimal control techniques, it is necessary to identify the state variables in each area. The number of such variables may be four or more depending upon the model chosen. Therefore the total number of state variables for a multiarea system may be quite large; and computation of optimal control for such a system not only presents storage problem on the computer but also other computational difficulties. It is therefore imperative that either some kind of approximation is adopted or some kind of suboptimal control is computed, to reduce the amount and time of computation. One such approximation technique is presented in this chapter. In Sections 2.2 to 2.4 reduction of a three-equal-area system to a two-unequal-area system using this technique, is considered; and in Sections 2.5 to 2.7 reduction of a three-unequal-area system is dealt with.

2.2 THREE-EQUAL-AREA SYSTEM

A three-equal-area system is so named because all the three areas are of the same Megawatt capacity and have identical gains and time constants in the corresponding control blocks. The choice of a three-equal-area system is first made for the sake of simplicity in computation. However, in Section 2.5 a more general case viz., a three-unequal-area system is taken up for study. Figure 2.1 gives a schematic diagram of a three-equal-area system. A step load disturbance is considered in area 1 to study the dynamic characteristics of the system. The three-areas are interconnected by means of tielines Tie_{12}, Tie_{13} and Tie_{23} respectively. The tieline angles are taken as 45°, 30° and 15° respectively. However, in a practical system these angles are determined not only by the angles of the voltages at either end of the tieline but also by the angles of the tieline impedances. Here, for simplicity, the tielines are taken to be purely reactive and thus the impedance angle is exactly 90°.

Each of the areas are represented by the state variables Δf, ΔX_{gv}, ΔP_g and ΔP_{tie}, representing the change in frequency, change in valve position as a result of governor action, change in steam turbine output and change in tieline power (in p.u.) flowing out of the area, respectively. In this three-area

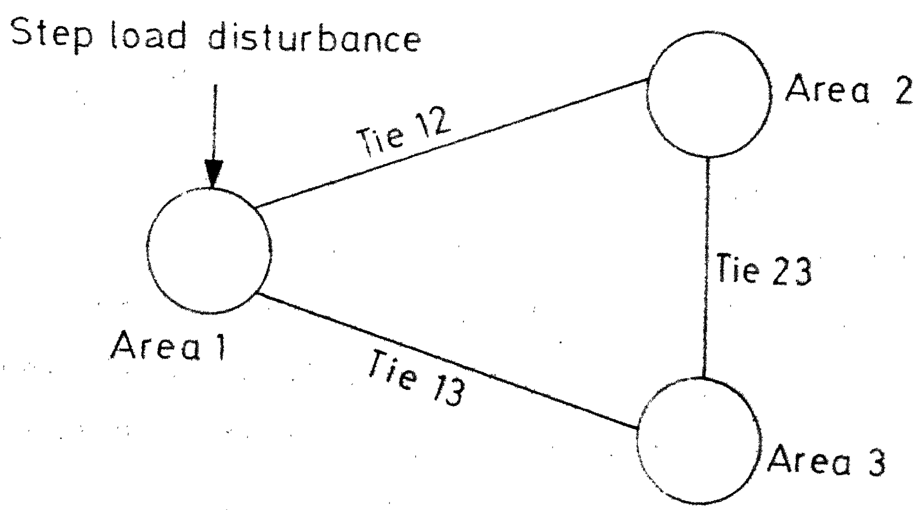


Fig. 2·1 Three equal-area system

system, it is enough to consider ΔP_{tie1} and ΔP_{tie2} only, because the tieline powers expressed in Megawatts, flowing out of the three-areas sum up to zero or

$$\Delta P_{tie3}\, P_{r3} = -(\Delta P_{tie1}\, P_{r1} + \Delta P_{tie2}\, P_{r2}) \tag{2.1}$$

where P_{r1}, P_{r2} and P_{r3} are the base powers in the three areas. Here, as the Megawatt capacity of the three-areas are equal, we have $P_{r1} = P_{r2} = P_{r3}$. Hence (2.1) reduces to

$$\Delta P_{tie3} = -(\Delta P_{tie1} + \Delta P_{tie2}) \tag{2.2}$$

It is assumed that the load disturbance in Area 1 is such that it results in only small variations in tie-line angles. Hence it is justified to take a linear system. The block diagram of a single area in a three-area system is given in Figure 2.2 and its mathematical description is given below:

$$\frac{2H_i}{f^*}\frac{d}{dt}\Delta f_i + D_i\,\Delta f_i + \Delta P_{tie\,i} = \Delta P_{gi} - \Delta P_{di} \tag{2.3}$$

$$\frac{d}{dt}\left(\int \Delta P_{tie\,i}\,dt\right) = \sum_{\substack{p=1\\ p\neq i}}^{3} T_{ip}\left(\int \Delta f_i\,dt - \int \Delta f_p\,dt\right) \tag{2.4}$$

$$\frac{d}{dt}\Delta P_{gi} = -\frac{1}{T_{ti}}\Delta P_{gi} + \frac{1}{T_{ti}}\Delta X_{gvi} \tag{2.5}$$

$$\frac{d}{dt}\Delta X_{gvi} = -\frac{1}{T_{gvi}}\Delta X_{gvi} - \frac{1}{T_{gvi}\,R_i}\Delta f_i + \frac{1}{T_{gvi}}\Delta P_{ci} \quad \ldots (2.6)$$

$$\ldots\; i = 1,2,3.$$

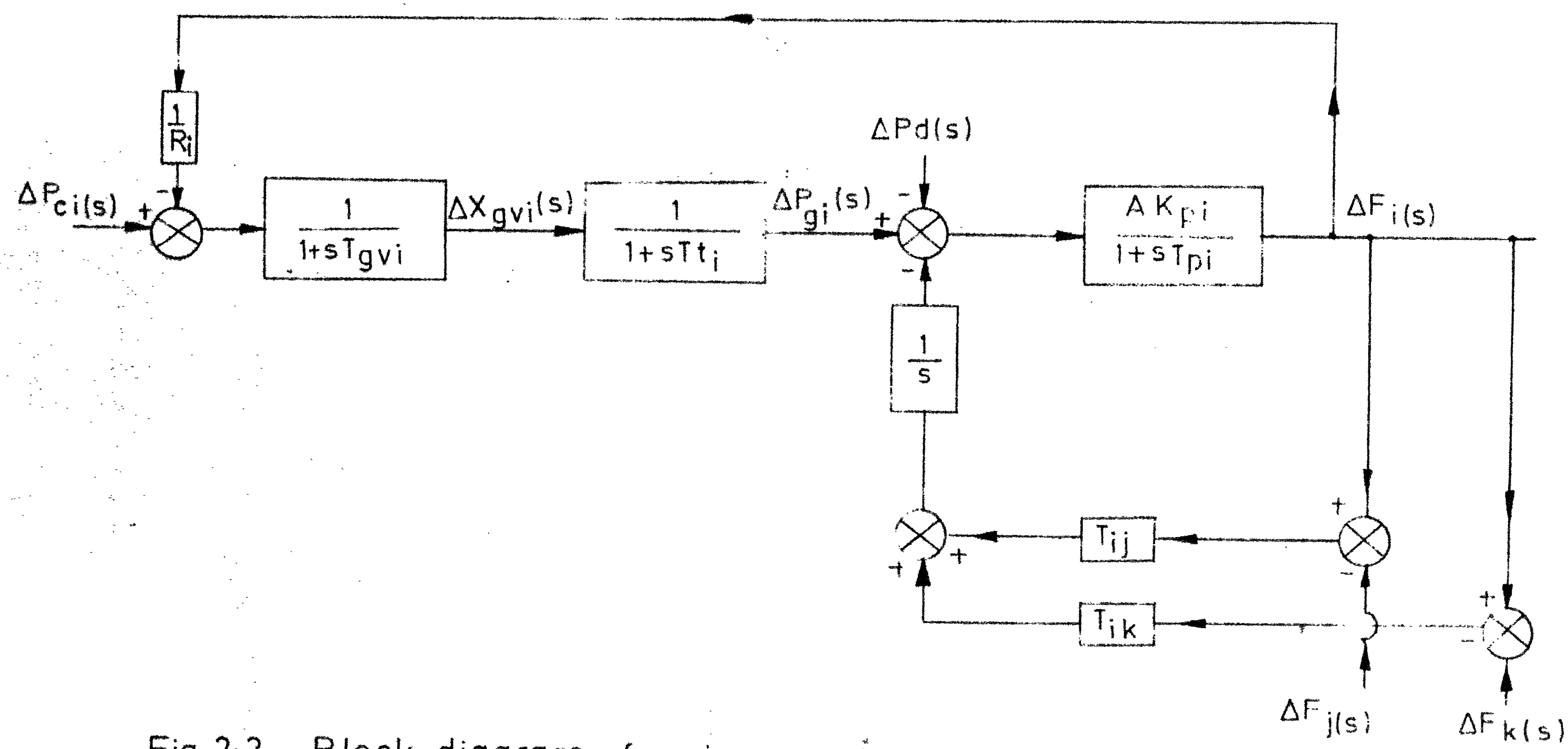


Fig. 2·2 Block diagram of a single area in a three area system

Parameters of the Three-area System

The description of the parameters used in the above differential equations and also their numerical values are given below:

f^* is the nominal frequency.

T_{t1}, T_{t2}, T_{t3} Steam turbine time constants in Areas 1,2 and 3.

$T_{gv1}, T_{gv2}, T_{gv3}$ Speed governor time constants.

R_1, R_2, R_3 Self regulation constants of generators.

D_1, D_2, D_3 Load frequency constants.

$P_{r1} = P_{r2} = P_{r3} = 2000$ MW

$$P_{tie\,1} = P_{tie\,12} + P_{tie13}$$

$$P_{tie1} = \frac{|V_1|\,|V_2|}{X_{12}\,P_{r1}} \operatorname{Sin}(\delta_1 - \delta_2) + \frac{|V_1|\,|V_3|}{X_{13}\,P_{r1}} \operatorname{Sin}(\delta_1 - \delta_3)$$

$$\Delta P_{tie1} = \Delta P_{tie12} + \Delta P_{tie13}$$

$$= \frac{|V_1|\,|V_2|}{X_{12}\,P_{r1}} \operatorname{Cos}(\delta_1^* - \delta_2^*)(\Delta\delta_1 - \Delta\delta_2)$$

$$+ \frac{|V_1|\,|V_3|}{X_{13}\,P_{r1}} \operatorname{Cos}(\delta_1^* - \delta_3^*)(\Delta\delta_1 - \Delta\delta_3)$$

$$= T_{12}(\int \Delta f_1 dt - \int \Delta f_2 dt) + T_{13}(\int \Delta f_1 dt - \int \Delta f_3 dt)$$

where

$$T_{12} = \frac{2\pi\,|V_1|\,|V_2|}{X_{12}\,P_{r1}} \operatorname{Cos}(\delta_1^* - \delta_2^*)$$

$$= 2\pi \times 0.1 \times \operatorname{Cos}(\delta_1^* - \delta_2^*)$$

and

$$T_{13} = \frac{2\pi\,|V_1|\,|V_3|}{X_{13}\,P_{r1}} \operatorname{Cos}(\delta_1^* - \delta_3^*)$$

$$= 2\pi \times 0.1 \times \operatorname{Cos}(\delta_1^* - \delta_3^*)$$

Also $\Delta\delta_1 = 2\pi \int \Delta f_1 dt$ and $\Delta\ \delta_2 = 2\pi \int \Delta f_2 dt$

ΔP_{tie2} and ΔP_{tie3} are defined in a manner similar to ΔP_{tie1}; T_{23} is defined in the same manner as T_{12}.

$$\delta_1^* - \delta_2^* = 45°, \quad \delta_1^* - \delta_3^* = 30°, \quad \delta_2^* - \delta_3^* = 15°$$

$f^* = 60$ Hz

$H_1 = H_2 = H_3 = 5$ sec.

$D_1 = D_2 = D_3 = 8.33 \times 10^{-3}$ p.u. MW/Hz

$T_{t1} = T_{t2} = T_{t3} = 0.3$ sec.

$T_{gv1} = T_{gv2} = T_{gv3} = 0.08$ sec.

$R_1 = R_2 = R_3 = 2.4$ Hz/p.u. MW

Also

$$A_{kpi} = \frac{1}{D_i} \quad \text{and} \quad T_{pi} = \frac{2H_i}{f^*\,D_i}, \qquad i = 1,2,3$$

The system described in (2.3) to (2.6) can be expressed dynamically as the vector differential equation

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \Gamma\,\Delta\,\underline{P}_d \tag{2.7}$$

Here $\underline{x}$ is an 11-element state vector consisting of the state variables Δf_1, Δf_2, Δf_3, ΔX_{gv1}, ΔX_{gv2}, ΔX_{gv3}, ΔP_{g1}, ΔP_{g2}, ΔP_{g3}, ΔP_{tie1} and ΔP_{tie2}; $\Delta\ \underline{P}_d$ is the load disturbance vector consisting of ΔP_{d1}, ΔP_{d2} and ΔP_{d3} as its elements.

For applying the optimal control theory, it is necessary to eliminate the third term in the right hand side of (2.7) thus reducing the equation to the familiar form:

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{2.8}$$

This has been achieved by redefining the states and controls in terms of their steady state values occurring after the disturbance is applied[3].

The performance index to be minimized is taken as

$$J = \frac{1}{2} \int_0^{\infty} (\underline{x}^T Q\,\underline{x} + \underline{u}^T R\,\underline{u})\,dt \tag{2.9}$$

Here R is taken to be a 3x3 identity matrix. The matrix Q is selected such that the frequency deviation from the nominal value is minimum in the three areas; also the tieline power deviation from the nominal value should be minimum for the three tielines.

The system matrices B, Γ, A and Q are described below:

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_{gv2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{gv3}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

... (2.10)

$$\Gamma = \begin{bmatrix} \frac{f^*}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{f^*}{2H_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{f^*}{2H_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad \ldots \quad (2.11)$$

$$A = \begin{bmatrix} \frac{-1}{T_{p1}} & 0 & 0 & 0 & 0 & 0 & \frac{A_{kp1}}{T_{p1}} & 0 & 0 & \frac{-A_{kp1}}{T_{p1}} & 0 \\ 0 & \frac{-1}{T_{p2}} & 0 & 0 & 0 & 0 & 0 & \frac{A_{kp2}}{T_{p2}} & 0 & 0 & \frac{-A_{kp2}}{T_{p2}} \\ 0 & 0 & \frac{-1}{T_{p3}} & 0 & 0 & 0 & 0 & 0 & \frac{A_{kp3}}{T_{p3}} & \frac{A_{kp3}}{T_{p3}} & \frac{A_{kp3}}{T_{p3}} \\ \frac{-1}{R_1 T_{gv1}} & 0 & 0 & \frac{-1}{T_{gv1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{R_2 T_{gv2}} & 0 & 0 & \frac{-1}{T_{gv2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{R_3 T_{gv3}} & 0 & 0 & \frac{-1}{T_{gv3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{t1}} & 0 & 0 & \frac{-1}{T_{t1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_{t2}} & 0 & 0 & \frac{-1}{T_{t2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{t3}} & 0 & 0 & \frac{-1}{T_{t3}} & 0 & 0 \\ T_{12}+T_{13} & -T_{12} & -T_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -T_{12} & T_{12}+T_{23} & -T_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\ldots \quad (2.12)$$

$$
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \quad \ldots (2.13)
$$

Pontryagin's minimum principle (optimal control theory) is applied to find the closed loop controller of the form (Appendix B)

$$\underline{u}(t) = -K\,\underline{x}(t) = -R^{-1}B^T P\,\underline{x}(t) \qquad (2.14)$$

where P is the solution of the matrix Riccati equation

$$\dot{P} = -P\,A - A^T P + P\,B\,R^{-1}\,B^T\,P - Q \qquad (2.15)$$

and is a 11x11 matrix. In view of infinite time involved $\dot{P}$ becomes zero and (2.15) becomes simply a matrix quadratic equation given by

$$0 = -P\,A - A^T P + P\,B\,R^{-1}\,B^T\,P - Q \qquad (2.16)$$

The following procedure is adopted for obtaining the solution of this equation. At first the differential

equation (2.15) is solved for 8 or 10 seconds such that the matrix P so obtained makes the closed loop system matrix $(A - B R^{-1} B^T P)$ as a stable matrix[20]. Then this result is fed as initial solution to the algorithm given by Man[11,12] and the correct solution matrix P is obtained. Appendix C gives a brief description of this algorithm of Man. The closed loop system matrix now becomes:

$$\dot{\underline{x}} = (A - B R^{-1} B^T P) \underline{x} \tag{2.17}$$

and if a load disturbance is applied in Area 1, this becomes

$$\dot{\underline{x}} = (A - B R^{-1} B^T P) \underline{x} + \Gamma \Delta \underline{P}_d \tag{2.18}$$

where Γ is a 11x3 load disturbance distribution matrix as described in (2.11). The vector

$$\Delta \underline{P}_d = \begin{bmatrix} \Delta P_{d1} \\ \Delta P_{d2} \\ \Delta P_{d3} \end{bmatrix} = \begin{bmatrix} 0.01 \\ 0.0 \\ 0.0 \end{bmatrix} \tag{2.19}$$

Equation (2.19) shows that a load disturbance of 0.01 p.u. of power is considered only in area 1. The initial condition vector is taken as $\underline{x}(o) = \underline{0}$. The dynamic equation (2.18) is solved and the responses of the state variables of interest viz., Δf_1, Δf_2, ΔP_{tie1} and Δf_3 are obtained. As the disturbance is in Area 1, the tie-line power interchange error of Area 1, ΔP_{tie1}, is of importance.

2.3 APPROXIMATE TWO-AREA SYSTEM

In Section 2.2, the three-equal-area system is studied in its complete shape without any approximations. In this section reduction in computation for the three-equal-area system is achieved by approximating the same as an equivalent two-area system. Here, the Areas 2 and 3 are combined together to form a single-area (Figure 2.3). As we have considered the three-areas to have identical parameters, the capacity in MW of the combined system 2 and 3 will be double that of any one of the three areas. The tieline here consists of two branches Tie_{12} and Tie_{13}. As our ultimate aim is to consider this system as a two-unequal-area system, the tielines Tie_{12} and Tie_{13} are combined together into a single equivalent tieline connecting the two-areas. The parameter T_{12} for this new system is approximated as the sum of the parameters T_{12}(old) and T_{13}(old) corresponding to Tie_{12} and Tie_{13} respectively. Thus

$$T_{12}(\text{new}) = T_{12}(\text{old}) + T_{13}(\text{old}) \tag{2.20}$$

or

$$\begin{aligned} T_{12}(\text{new}) &= 0.1 \times \text{Cos}45^\circ \times 2\pi + 0.1 \times \text{Cos}30^\circ \times 2\pi \\ &= 0.1 \times 2\pi \times (\text{Cos } 30^\circ + \text{Cos } 45^\circ) \end{aligned} \tag{2.21}$$

This may also be interpreted as taking the new tieline to consist of two component tielines, the angle for both of which is the same and is approximated as the average of the angles of the corresponding tielines of the three-area-system.

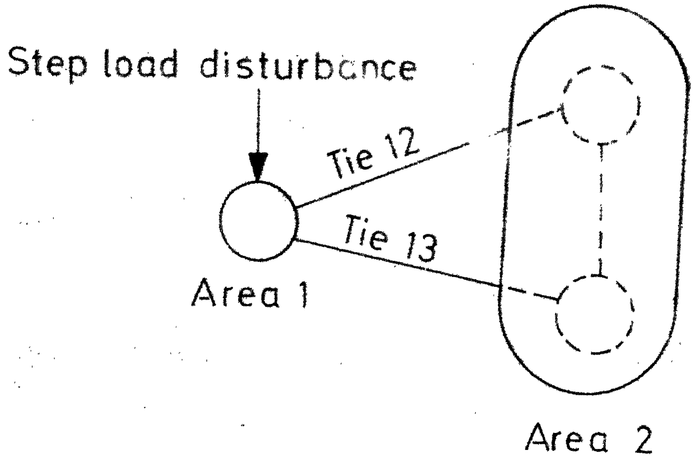


Fig. 2·3 Equivalent two-area system

The relationship between ΔP_{tie2} and ΔP_{tie1} becomes

$$\Delta P_{tie2} = a_{12} \ \Delta P_{tie1} \tag{2.22}$$

where ΔP_{tie1} and ΔP_{tie2} are in per unit of power. As their MW equivalents are equal and opposite in sign,

$$\Delta P_{tie2} \ P_{r2} = - \Delta P_{tie1} \ P_{r1} \tag{2.23}$$

where P_{r1} and P_{r2} are the base powers in the new Areas 1 and 2 respectively. So

$$\Delta P_{tie2} = -\left(\frac{P_{r1}}{P_{r2}}\right) \ \Delta P_{tie1} \tag{2.24}$$

Because we have combined the areas 2 and 3 of the three-equal-area-system, the base power of Area 2 in the new configuration will be twice that of either one of the areas. Thus

$$P_{r2} = 2P_{r1} \tag{2.25}$$

From (2.22), (2.24) and (2.25), we get

$$a_{12} = -\left(\frac{P_{r1}}{P_{r2}}\right) = -0.5 \tag{2.26}$$

Because of this relation, we need not consider ΔP_{ie2} as a a state variable. For the equivalent two area system, the system vector differential equation becomes

$$\dot{\underline{x}} = A \ \underline{x} + B \ \underline{u} \tag{2.27}$$

where $\underline{x}$ is a 7-element vector consisting of the state variables Δf_1, Δf_2, ΔX_{gv1}, ΔX_{gv2}, ΔP_{g1}, ΔP_{g2} and ΔP_{tie1}.

The matrices A(7x7) and B(7x2) are given by

$$A = \begin{bmatrix} -\frac{1}{T_{p1}} & 0 & 0 & 0 & \frac{A_{kp1}}{T_{p1}} & 0 & -\frac{A_{kp1}}{T_{p1}} \\ 0 & -\frac{1}{T_{p2}} & 0 & 0 & 0 & \frac{A_{kp2}}{T_{p2}} & -\frac{A_{kp2}}{T_{p2}}a_{12} \\ -\frac{1}{R_1 T_{gv1}} & 0 & -\frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_2 T_{gv2}} & 0 & -\frac{1}{T_{gv2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_{t1}} & 0 & -\frac{1}{T_{t1}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{t2}} & 0 & -\frac{1}{T_{t2}} & 0 \\ T_{12} & -T_{12} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \ldots \quad (2.28)$$

$$B = \begin{bmatrix} 0 & 0 & \frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{gv2}} & 0 & 0 & 0 \end{bmatrix}^T \qquad \ldots \quad (2.29)$$

All the parameters used for the elements of the matrices A and B are as given in Section 2.2 except for the value of a_{12} which is given in (2.26).

The performance index to be minimized is

$$J = \frac{1}{2} \int_0^{\infty} (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u}) \, dt \qquad (2.30)$$

The matrix Q is selected taking into consideration the fact that the state variables Δf_1, Δf_2(new) and ΔP_{tie1}

should have minimum variation. Thus

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.31)$$

R is taken as a 2x2 identity matrix. Pontryagin's minimum principle (Appendix B) is applied to get the control vector $\underline{u}$ as

$$\underline{u} = -R^{-1}\, B^T\, P\, \underline{x} \qquad (2.32)$$

where P is the solution of the matrix Riccati equation

$$\dot{P} = -P\,A - A^T P + P\,B\,R^{-1}\,B^T\,P - Q \qquad (2.33)$$

Here P is a 7x7 matrix. The responses of the state variables Δf_1, Δf_2, and ΔP_{tie1} are determined by solving the closed loop vector differential equation with a step load disturbance of 0.01 p.u. of power in Area 1. For this purpose, (2.27) is augmented with load disturbance as

$$\dot{\underline{x}} = (A - B\,R^{-1}\,B^T\,P)\,\underline{x} + \Gamma\,\Delta\underline{P}_d \qquad (2.34)$$

Here Γ is a 7x2 load-disturbance-distribution matrix.

$$\Gamma = \begin{bmatrix} \frac{f^*}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{f^*}{2H_2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad (2.35)$$

and

$$\Delta \underline{P}_d = \begin{bmatrix} \Delta P_{d1} \\ \Delta P_{d2} \end{bmatrix} \quad \begin{bmatrix} 0.01 \\ 0.0 \end{bmatrix} \tag{2.36}$$

Equation (2.36) shows that load disturbance is considered to occur in the same area in both the three-area and two-area cases. Thus the responses obtained in both cases can be compared to give an idea about the efficacy of the approximation.

2.4 COMPUTATIONAL RESULTS FOR THREE-EQUAL-AREA SYSTEM

(a) Three-area System (11th order)

The B matrix (2.10) is computed as

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 12.5 & 0 & 0 \\ 0 & 12.5 & 0 \\ 0 & 0 & 12.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix}
-0.04998 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 & 0.0 \\
0.0 & -0.04998 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 \\
0.0 & 0.0 & -0.04998 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 6.0 & 6.0 \\
-5.20833 & 0.0 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -5.20833 & 0.0 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -5.20833 & 0.0 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 3.33333 & 0.0 & 0.0 & -3.33333 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 3.33333 & 0.0 & 0.0 & -3.33333 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.33333 & 0.0 & 0.0 & -3.33333 & 0.0 & 0.0 \\
0.98879 & -0.44446 & -0.54433 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.44446 & 1.05134 & -0.60688 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}$$

The feedback matrix $(A-B\,R^{-1}\,B^{T}\,P)$ (2.17) is computed as

$$\begin{bmatrix}
-0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 & 0.0 \\
0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 \\
0.0 & 0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 6.0 & 6.0 \\
-13.226 & 0.305 & -0.431 & -15.535 & 0.139 & -0.016 & -12.765 & 0.645 & -0.070 & 12.130 & 0.687 \\
0.317 & -13.200 & -0.469 & 0.139 & -15.539 & -0.012 & 0.649 & -12.683 & -0.056 & 0.832 & 12.226 \\
-0.442 & -0.457 & -12.452 & -0.016 & -0.012 & -15.384 & -0.074 & -0.052 & -12.063 & -12.963 & -12.913 \\
0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 \\
0.989 & -0.444 & -0.544 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.444 & 1.051 & -0.607 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}$$

The responses of the state variables Δf_1, Δf_2 are shown as continuous curves in Figure 2.4 and that for ΔP_{tie1} is shown as continuous curve in Figure 2.5. The response of Δf_3 is found to be almost identical to that of Δf_2 and hence not given.

b) Two-area System (7th order)

The A matrix of (2.28) is computed as

$$\begin{bmatrix} -0.05 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & -6.0 \\ 0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 6.0 & 3.0 \\ -5.208 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -5.208 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 \\ 0.989 & -0.989 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

The B matrix of (2.29) is computed as

$$\begin{bmatrix} 0 & 0 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12.5 & 0 & 0 & 0 \end{bmatrix}^T$$

The closed loop matrix $(A - B\,R^{-1}\,B^T\,P)$ of (2.34) is computed as

$$\begin{bmatrix} -0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -6.0 \\ 0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 6.0 & 3.0 \\ -12.43 & -0.884 & -15.35 & 0.141 & -11.89 & 0.339 & 13.01 \\ 1.129 & -14.52 & 0.141 & -15.75 & 0.978 & -13.78 & -3.96 \\ 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 \\ 0.989 & -0.989 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

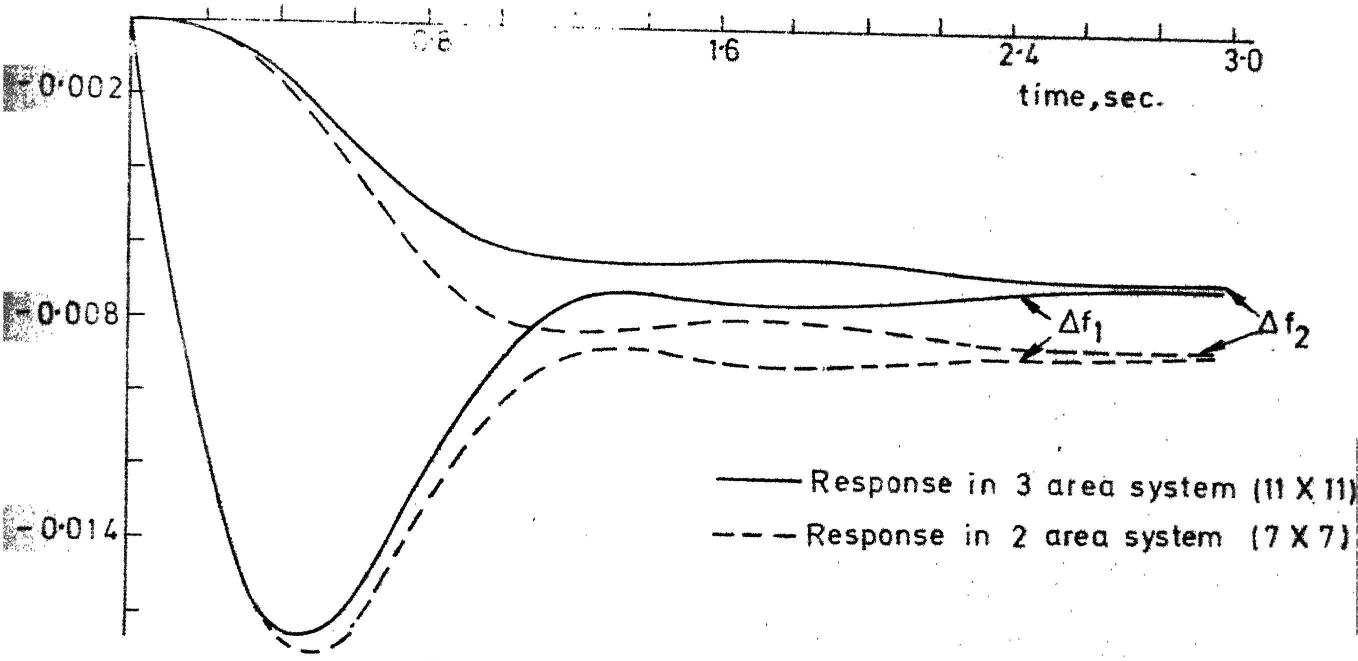


Fig. 2·4 Response of Δf_1 and Δf_2

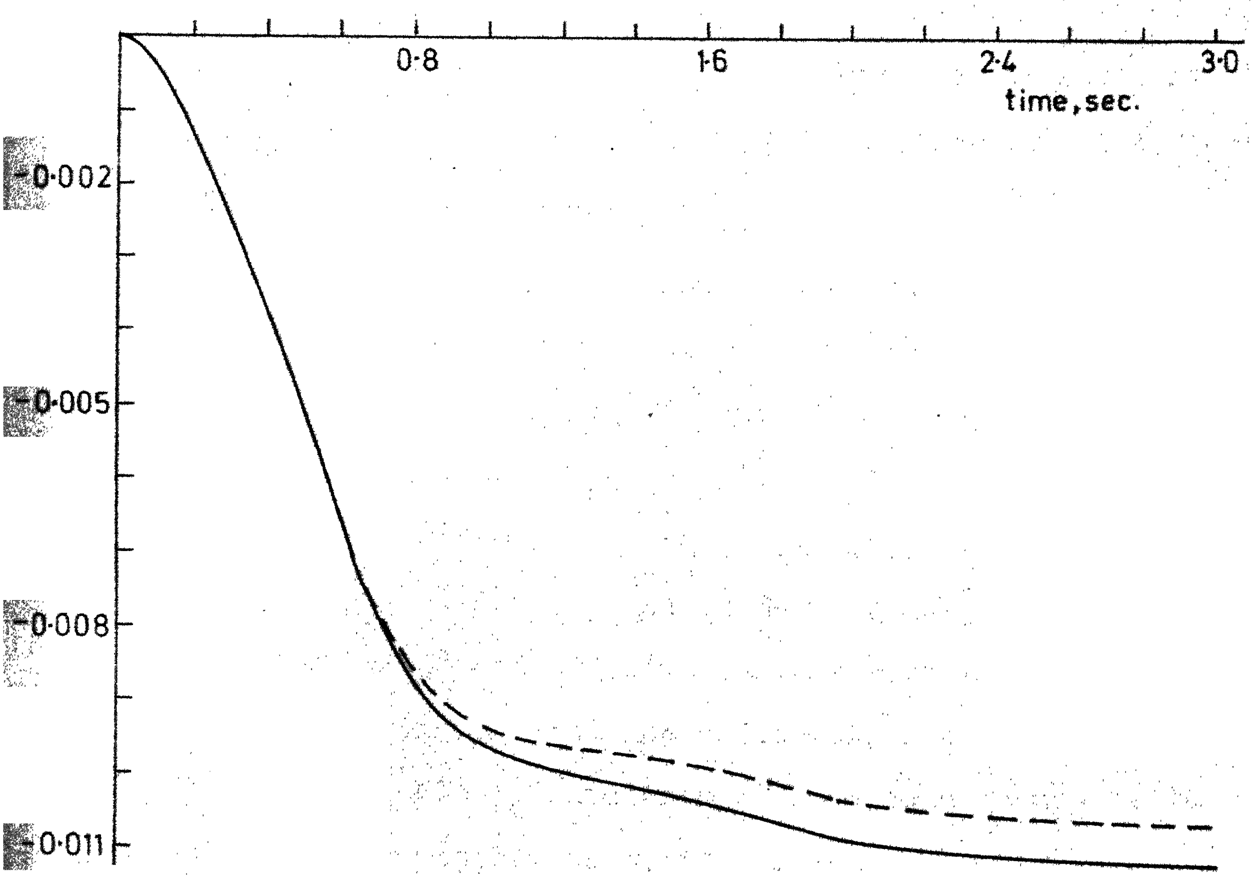


Fig. 2·5 Response of $\Delta P_{tie\ 1}$

The responses of the state variables Δf_1 of the smaller area (Area 1) and Δf_2 of the larger area (Area 2) are given as dashed curves in Figure 2.4. Also ΔP_{tie1}, the total tieline power change out of Area 1 is given as dashed curve in Figure 2.5.

2.5 THREE-UNEQUAL-AREA SYSTEM

As is evident from the title, here the three-areas are considered to have different base powers and correspondingly different parameters. However for simplicity of computation, per unit values of the parameters like inertia constant H, load frequency constant D, self regulation constant R are all considered to be equal in the three areas. There is no loss of generality in this way of treatment because what is more important is consideration of different base powers in the three areas.

In Figure 2.6 is given a schematic diagram of a three-unequal-area system in which the MW capacity of Area 2 is 1.2 times the MW capacity of Area 1 and the MW capacity of Area 3 is 1.5 times that of Area 1. The tieline angles for the three tielines Tie_{12}, Tie_{13} and Tie_{23} are taken as 45°, 30° and 15° respectively as in Section 2.2. The state space analysis of this 11 state variable system is the same as in Section 2.2 except that the differential equations pertaining to Δf_3 and ΔP_{tie2} are changed as a consequence of the different MW capacities of the areas.

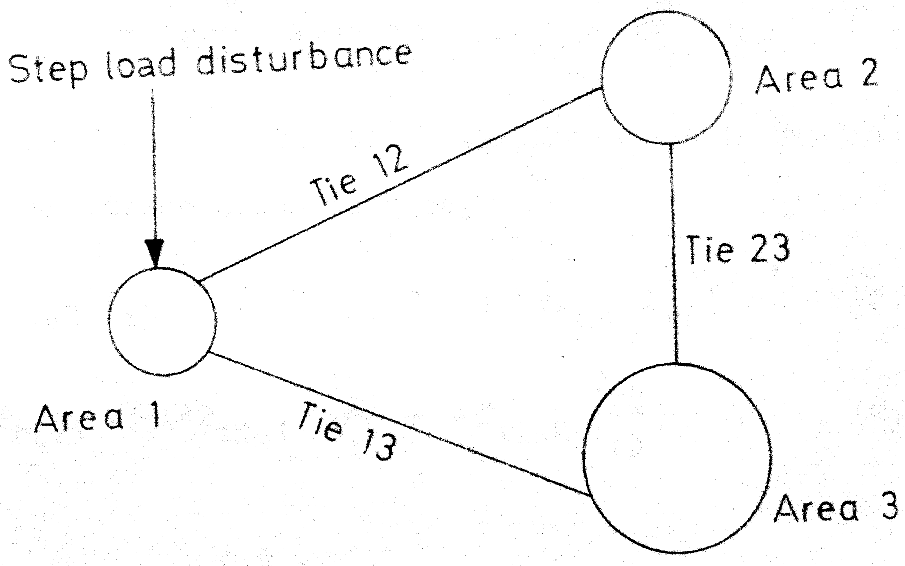


Fig. 2·6 Three unequal-area system

(i) Δf_3:

$$\frac{2H_3}{f^*}\frac{d}{dt}\Delta f_3 = -D_3\Delta f_3 - \Delta P_{tie3} + \Delta P_{g3} - \Delta P_{d3} \qquad (2.37)$$

Considering $\Delta P_{d3} = 0.0$ and rewriting the above equation,

$$\frac{d}{dt}\Delta f_3 = -\frac{f^* D_3}{2H_3}\Delta f_3 - \frac{f^*}{2H_3}\Delta P_{tie3} + \frac{f^*}{2H_3}\Delta P_{g3} \qquad (2.38)$$

Using the fact that the total tieline power in MW flowing out of the three areas is zero,

$$\Delta P_{tie3}\, P_{r3} = -(\Delta P_{tie1}\, P_{r1} + \Delta P_{tie2}\, P_{r2}) \qquad (2.39)$$

or

$$\Delta P_{tie3} = -(\Delta P_{tie1}\frac{P_{r1}}{P_{r3}} + \Delta P_{tie2}\frac{P_{r2}}{P_{r3}}) \qquad (2.40)$$

Here

$$\frac{P_{r1}}{P_{r3}} = \frac{1}{1.5} \quad \text{and} \quad \frac{P_{r2}}{P_{r3}} = \frac{1.2}{1.5}$$

Hence

$$\Delta P_{tie3} = -(\Delta P_{tie1}\frac{1}{1.5} + \Delta P_{tie2}\frac{1.2}{1.5}) \qquad (2.41)$$

Substituting this value for ΔP_{tie3} in (2.38)

$$\frac{d}{dt}\Delta f_3 = -\frac{f^* D_3}{2H_3}\Delta f_3 + \frac{f^*}{2H_3}\Delta P_{g3} + \frac{f^*}{2H_3}\frac{\Delta P_{tie1} \times 1}{1.5}$$

$$+ \frac{f^*}{2H_3}\Delta P_{tie2} \times \frac{1.2}{1.5} \qquad (2.42)$$

Thus the elements A(3, 10) and A(3, 11) of the 11x11 system matrix A in (2.12) are modified as

$$A(3,10) = \frac{f^*}{2H_3} \times \frac{1}{1.5} = \frac{A_{kp3}}{T_{p3}} \times \frac{1}{1.5}$$

and

$$A(3,11) = \frac{f^*}{2H_3} \times \frac{1.2}{1.5} = \frac{A_{kp3}}{T_{p3}} \times \frac{1.2}{1.5}$$

the rest of the elements of the 3rd row remaining the same.

(ii) ΔP_{tie2}:

$$\frac{d}{dt}(\Delta P_{tie2}) = T_{21}(\Delta f_2 - \Delta f_1) + T_{23}(\Delta f_2 - \Delta f_3) \quad (2.43)$$

Here

$$T_{21} = T_{12} \times \frac{P_{r1}}{P_{r2}} = T_{12} \times \frac{1}{1.2} \quad (2.44)$$

where T_{12} is expressed in p.u. of base power P_{r1}. T_{23} remains the same provided it is expressed in p.u. of P_{r2} only. Hence the elements A(11, 1) and A(11, 2) of the matrix A in (2.12) are modified as

$$A(11, 1) = -T_{12} \times \frac{1}{1.2} \quad (2.45)$$

and

$$A(11,2) = (T_{12} \times \frac{1}{1.2} + T_{23}) \quad (2.46)$$

the rest of the elements in the 11th row remaining the same as given in (2.12). Also all the elements in the other rows except 3 and 11 remain unchanged.

In this three-unequal-area-system the following values are assumed for T_{12}, T_{13} and T_{23}:

$T_{12} = 0.1 \times \cos 45° \times 2\pi$ expressed in p.u. of P_{r1}

$T_{13} = 0.1 \times \cos 30° \times 2\pi$ expressed in p.u. of P_{r1}

$T_{23} = 0.1 \times \cos 15° \times 2\pi$ expressed in p.u. of P_{r2}.

The matrices B, Q, R and also the theory and analysis remain the same as in Section 2.2.

A step load disturbance of 0.01 p.u. of power is considered in Area 1 and the responses of the state variables of interest viz. Δf_1, Δf_2 and ΔP_{tie1} are obtained.

2.6 APPROXIMATE TWO-UNEQUAL-AREA SYSTEM

Here two of the areas in the three-area system are combined together as is done in Section 2.3, thus converting the three-area system into an equivalent two-area system. The system matrix A is having the same elements as in (2.28) except for the following slight modifications.

As we have to combine the Areas 2 and 3 as a composite area, their combined capacity (and inertia) is 2.7 times the capacity (and inertia) of Area 1. Hence

$$\Delta P_{tie2} = -\frac{P_{r1}}{P_{r2}} \Delta P_{tie1} = -\frac{1}{2.7} \times \Delta P_{tie1} \qquad (2.47)$$

In the differential equation pertaining to Δf_2 the value of a_{12} is taken as $(-1/2.7)$ in view of the above relation. Thus only the element A(2,7) of the matrix A in (2.28) is changed, the rest of the elements remaining unchanged.

The matrices B, Q, R as well as the theory and analysis remain the same as given in Section 2.3.

A step load disturbance of 0.01 p.u. of power is considered in Area 1 and the responses of Δf_1, Δf_2(new) and ΔP_{tie1} are obtained as before.

2.7 COMPUTATIONAL RESULTS FOR THREE-UNEQUAL-AREA SYSTEM

(a) Three-Unequal-Area System (11th order)

The closed loop matrix $(A - BK)$, 11x11 in this case is computed as

$$\begin{bmatrix}
-0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 & 0.0 \\
0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 0.0 & -6.0 \\
0.0 & 0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 4.0 & 4.8 \\
-13.025 & 0.268 & -0.587 & -15.487 & 0.160 & 0.021 & -12.544 & 0.696 & -0.020 & 12.413 & 0.826 \\
0.589 & 13.197 & -0.744 & 0.160 & -15.525 & -0.025 & 0.798 & -12.723 & -0.202 & 1.373 & 12.482 \\
0.161 & 0.224 & -13.297 & 0.021 & -0.025 & -15.532 & 0.218 & -0.036 & -12.750 & -7.611 & -9.650 \\
0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & 0.0 & -3.333 & 0.0 & 0.0 \\
0.989 & -0.444 & -0.544 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.370 & 0.977 & -0.607 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}$$

The responses obtained for Δf_1 and Δf_2 in this case are given as continuous curves in Figure 2.7 and that obtained for ΔP_{tie1} is shown as continuous curve in Figure 2.8.

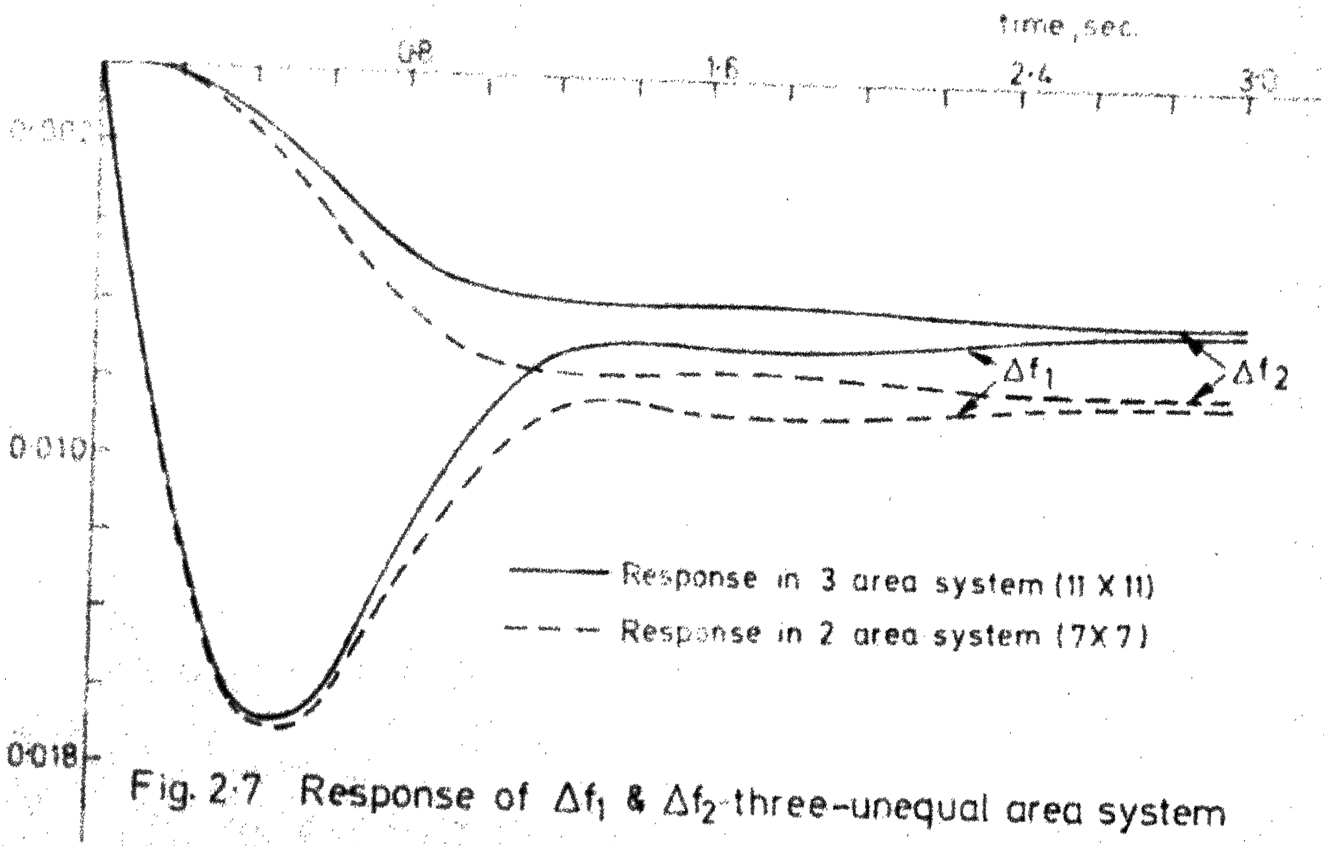


Fig. 2·7 Response of Δf_1 & Δf_2-three-unequal area system

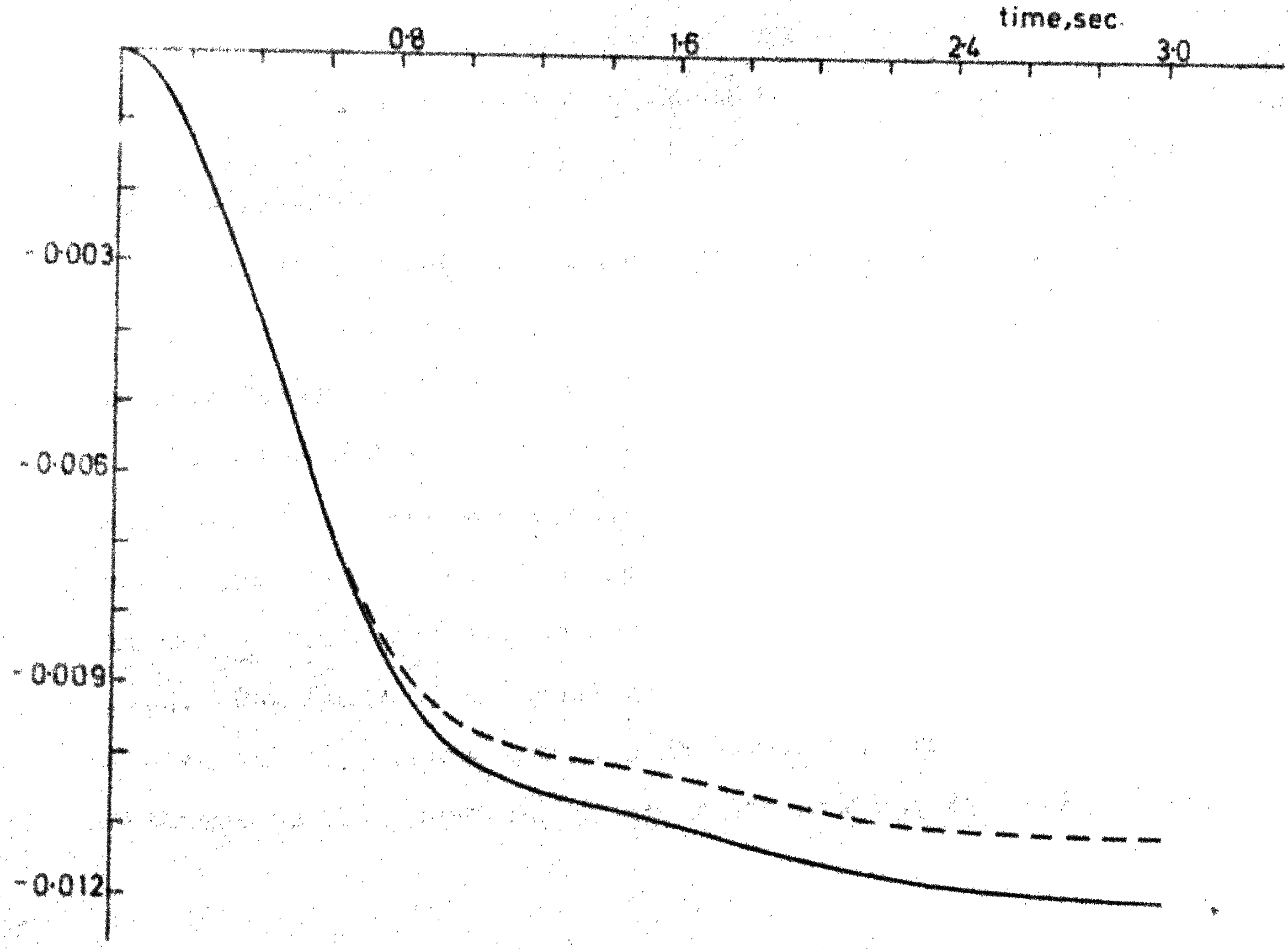


Fig 2·8 Response of ΔP_{tie1}-three-unequal-area system

(b) <u>Equivalent Two-Area System</u>

The closed loop 7x7 matrix(A-BK) for the equivalent two-area system is computed as

$$\begin{bmatrix} -0.050 & 0.0 & 0.0 & 0.0 & 6.0 & 0.0 & 6.0 \\ 0.0 & -0.05 & 0.0 & 0.0 & 0.0 & 6.0 & 2.222 \\ -12.403 & -0.906 & -15.339 & 0.186 & -11.859 & 0.462 & 13.025 \\ 1.654 & -15.050 & 0.185 & -15.844 & 1.274 & -14.222 & -1.360 \\ 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.333 & 0.0 & -3.333 & 0.0 \\ 0.989 & -0.989 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

The responses obtained for Δf_1 and Δf_2 in this case are shown as dashed curves in Figure 2.7 and that obtained for ΔP_{tie1} is shown as dashed curve in Figure 2.8.

2.8 CONCLUSIONS

In this chapter, a three-equal-area system is first taken up and approximated as a two-unequal-area system. Optimal controllers are constructed and responses obtained for Δf_1, Δf_2 and ΔP_{tie1} in both cases. Figures 2.4 and 2.5 show that the responses obtained with the two-area representation are very near to the optimal responses. Hence it can be said that the two-area approximation is quite good. The study of a three-unequal-area system and its equivalent representation is next taken up. The responses obtained in this case and given in Figures 2.7 and 2.8

also confirm the above results. It is seen from the above four figures that there is a small error in the steady state value in each of the responses obtained with the two-area representations.

The study made in this chapter indicates that the approximation can be done, for four-area and other higher order multi-area systems with good results. It is necessary that the area in which the disturbance occurs, be singled out as a separate area in the reduced system also, as otherwise the approximation may not prove to be advantageous.

In the three-area as well as the equivalent two-area systems studied above, integral feedback is absent, i.e., the states corresponding to $\int \Delta f dt$, $\int \Delta P_{tie} dt$ of the individual areas are absent. Hence the feedback control in all the cases, is unable to bring back the response of the state variables to zero steady state. Evidently, if integral feedback is also present, the responses ultimately go to zero steady state. Hence there will not be any steady state error and the approximation should prove to be much better.

The saving in computational effort as well as in computational time accruing out of such approximation is considerable. This could be seen from the fact that for a 11th order system (three-area system) matrix Riccati equation of 66th order is to be solved; whereas for a 7th order system (equivalent two-area system) the order of the matrix Riccati equation to be solved is only 28.

CHAPTER 3

SUBOPTIMAL CONTROL USING AGGREGATION

In Chapter 2, reduction in computation of linear multiarea LFC systems was achieved by approximating them as lesser order multiarea systems. Here a different approach is attempted with the same goal in mind. For this purpose, the theory of aggregation due to Aoki[6] is applied.

In Sections 3.2 and 3.3 the theory of aggregation and suboptimal control by aggregation[6] are reviewed beiefly. Sections 3.4 and 3.5 deal with a method of computing optimal and suboptimal performance indices. In Section 3.6 determination of closed loop stability is discussed; and finally the computational results are presented in Sections 3.7 to 3.10.

3.2 THEORY OF AGGREGATION[6]

Let the dynamics of the system which is linear, be given by the vector differential equation

$$\dot{\underline{x}} = A\underline{x} + B\,\underline{u} \tag{3.1}$$

Here $\underline{x}$ is an n-vector

$\underline{u}$ is an r-vector

A is an nxn matrix

B is an nxr matrix.

Let $\underline{z}$ (an m vector, $m < n$), called the aggregated state vector be related to $\underline{x}$ by the relation

$$\underline{z}(t) = C\underline{x}(t) \tag{3.2}$$

where C is an mxn matrix and its m rows are the m dominant eigenvectors of A^T. It is assumed that the rank of C = m.

Assuming certain matrix equalities (given below) to be satisfied, the response of the above system can be approximately determined knowing the optimal feedback coefficients for the reduced system, viz.,

$$\dot{\underline{z}} = F\,\underline{z} + G\,\underline{u} \tag{3.3}$$

with the assumption that $\underline{z}(o) = C\,\underline{x}(o)$.

Here F and G are related to A and B by

$$F\,C = C\,A \tag{3.4}$$

and

$$G = C\,B \tag{3.5}$$

If A and C satisfy the matrix equation

$$C\,A = C\,A\,C^T(C\,C^T)^{-1}\,C \tag{3.6}$$

then F in (3.4) is given by

$$F = C\,A\,C^T(C\,C^T)^{-1} \tag{3.7}$$

The aggregated state vector $\underline{z}$ satisfies the dynamic equation (3.3) with F given in (3.7).

Here C is called the aggregation matrix and F, the aggregation of A. C is the important design parameter in constructing the reduced or aggregated system. The choice of C is to be made in such a way that error in modelling

the original system by means of the reduced system is minimized. To accomplish this, it is necessary that the matrix C be selected such that its m rows are the m eigenvectors of the transpose of the A matrix, corresponding to the m dominant eigenvalues. Moreover, F is chosen such that it retains the dominant eigenvalues of A; mathematically this means that F should satisfy (3.7).

3.3 APPLICATION TO DETERMINE SUBOPTIMAL CONTROL

Let the performance index of the original system be given by

$$J = \int_0^\infty (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u})\, dt \tag{3.8}$$

where Q is positive semidefinite and R is positive definite. The reduced mth order system is given by (3.3) with the initial condition $\underline{z}(o) = C\,\underline{x}(o)$.

Let $\underline{u} = -K\,\underline{z}$ be the optimal control policy for the reduced system where

$$K = R^{-1} G^T P \tag{3.9}$$

and P is the solution of the algebraic matrix Riccati equation

$$F^T P + P F - P G R^{-1} G^T P + Q_M = 0 \tag{3.10}$$

Here Q_M is defined as in (3.12) below. The smaller system is considered to have the following performance index with Q_M as the weighting matrix for the reduced states.

$$J_M = \int_0^\infty (\underline{z}^T Q_M \underline{z} + \underline{u}^T R \underline{u}) \, dt \tag{3.11}$$

where the appropriate choice of Q_M is

$$Q_M = (C\,C^T)^{-1} C\,Q\,C^T (C\,C^T)^{-1} \tag{3.12}$$

With this choice of Q_M,

$$\underline{u} = -K\,\underline{z} = -K\,C\,\underline{x} \tag{3.13}$$

becomes the suboptimal control, for the original system, which very nearly approximates the optimal control. With the above suboptimal control, the original system is governed by the equation

$$\dot{\underline{x}} = (A - B\,K\,C)\,\underline{x} \tag{3.14}$$

The optimal feedback system is given by

$$\dot{\underline{x}}^* = (A - B\,K^*)\,\underline{x}^* \tag{3.15}$$

In (3.15) the optimal control is given as

$$\underline{u}^* = -K^*\,\underline{x}^* \tag{3.16}$$

where

$$K^* = R^{-1}\,B^T\,T^* \tag{3.17}$$

and T^* satisfies

$$A^T\,T^* + T^*\,A - T^*\,B\,R^{-1}\,B^T\,T^* + Q = \underline{0} \tag{3.18}$$

3.4 MEASURE OF PERFORMANCE INDEX WITH OPTIMAL AND SUBOPTIMAL CONTROLS

The following is a brief summary of the results of Levine and Athans[15] who have given a method for obtaining a measure of the performance index with optimal and suboptimal controls.

Ths closed loop system, after incorporating the sub-optimal control given by (3.13) becomes

$$\underline{\dot{x}}(t) = (A - B\,K\,C)\,\underline{x}(t) \tag{3.19}$$

The solution of this differential equation becomes

$$\underline{x}(t) = \phi(t,\ 0)\ \underline{x}(o) \tag{3.20}$$

where $\phi(t,\ 0) = \exp([A - B\,K\,C]\ t)$ (3.21)

Substituting (3.20) into the performance criterion given in (3.8),

$$J = \underline{x}^T(o) \left[\frac{1}{2} \int_0^\infty \phi^T(t,\ 0)\ (Q + C^T K^T RKC)\phi(t,0)dt \right] x(o) \qquad \ldots (3.22)$$

Equation (3.22) shows that the performance criterion depends on K, C and the initial condition $\underline{x}(o)$. We can eliminate the dependence on $\underline{x}(o)$ by averaging the performance obtained for a linearly independent set of initial states which is equivalent to choosing the initial state x(o) to be a random variable uniformly distributed on the surface of the n-dimensional unit sphere. Then the expected value $\hat{J}$ of J becomes

$$\hat{J} = \frac{1}{2n} \int_0^\infty \mathrm{tr} \left[\phi^T(t,0)(Q+C^T K^T RKC)\phi(t,0) \right] dt \tag{3.23}$$

$\hat{J}$ is now independent of the initial state and can also be interpreted as a completely deterministic performance criterion in which the control consists of product of K and C and the state is $\phi(t,0)$ the fundamental transition matrix. In (3.23) we can remove n which is only a constant.

The optimization problem now becomes:

Given the performance index

$$\hat{J} = \frac{1}{2} \int_0^{\infty} \mathrm{tr}\left[\phi^T(t,0)(Q+C^T K^T RKC)\ \phi(t,0)\right] dt \qquad (3.24)$$

Find K which minimizes the performance criterion (3.24) subject to the constraint imposed by the system

$$\dot{\phi}(t,0) = \left[A - B\ K\ C\right] \phi(t,0)\ , \quad \phi(0,0) = I \qquad (3.25)$$

The main result of a theorem proved by Levine and Athans[15] for such a problem is as follows:

Let

$$H = (A - B\ K\ C) \qquad (3.26)$$

Assuming that H is stable, then in order for K to be optimal for the original system, it is necessary that

$$K = R^{-1}\ B^T\ T\ V\ C^T \left[C\ V\ C^T\right]^{-1} \qquad (3.27)$$

where

$$T = \int_0^{\infty} \exp(H^T \tau)\ (Q + C^T K^T RKC)\exp(H\tau)d\tau \qquad (3.28)$$

$$V = \int_0^{\infty} \exp(H\sigma)\ \exp\ (H^T\sigma)\ d\sigma \qquad (3.29)$$

Alternatively, assuming that K, T, and V are solutions of (3.27) to (3.29) respectively then T is also a positive definite solution of

$$0 = T\ H + H^T\ T + Q + C^T\ K^T\ RKC \qquad (3.30)$$

and V is a positive definite solution of

$$0 = V\ H^T + H\ V + I \qquad (3.31)$$

Also if we assume that C^{-1} exists and denoting T under this condition as T* (3.27) reduces to

$$K = R^{-1} B^T T^* C^{-1} \tag{3.32}$$

and (3.30) reduces to

$$0 = T^*A + A^T T^* + Q - T^* B R^{-1} B^T T^* \tag{3.33}$$

which is the same as (3.18).

3.5 DEGRADATION IN PERFORMANCE INDEX WITH SUBOPTIMAL CONTROL

From (3.24), (3.28) and (3.30) it is clear that the trace of the matrix T is a measure of the performance using suboptimal control given by (3.13), irrespective of initial conditions;also the trace of the matrix T* gives a measure of the performance index using optimal control.

For the load frequency control problem under study, the matrix T as given by (3.30) for the two cases of aggregation considered (viz. 6th order and 4th order), and the matrix T* as given by (3.18) are computed. The traces of the matrices T and T* are then computed and compared to assess the degradation in performance index.

3.6 STABILITY OF THE CLOSED LOOP SYSTEM WITH SUBOPTIMAL CONTROL

The stability of the closed-loop system using suboptimal control is determined by directly computing the eigenvalues of the closed-loop matrix (A - B K C) for the two cases of aggregation. However, this could also be

determined by Liapunov's equation.[21] If the solution matrix $\underset{\sim}{M}$, which is the solution of the matrix equation

$$(\underset{\sim}{A} - \underset{\sim}{B}\,\underset{\sim}{K}\,\underset{\sim}{C})^T \underset{\sim}{M} + \underset{\sim}{M}(\underset{\sim}{A} - \underset{\sim}{B}\,\underset{\sim}{K}\,\underset{\sim}{C}) = -\underset{\sim}{N} \tag{3.34}$$

is a positive definite matrix with the matrix $\underset{\sim}{N}$ being arbitrarily chosen to be positive definite, then the closed loop system matrix $(\underset{\sim}{A} - \underset{\sim}{B}\,\underset{\sim}{K}\,\underset{\sim}{C})$ is stable.

3.7 COMPUTATION OF SUBOPTIMAL CONTROLLERS FOR 6TH ORDER AGGREGATION

The A and B matrices for the load frequency control system are as defined in (A.8) and (A.9) of Appendix A and computed as

$$A = \begin{bmatrix} 0.0 & 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.27 & -0.05 & 6.0 & 0.0 & 3.27 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -3.333 & 3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -5.208 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 3.27 & 0.0 & 0.0 & 0.0 & -3.27 & -0.05 & 6.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.333 & 3.333 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -5.208 & 0.0 & -12.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.5 \end{bmatrix}^T$$

Here the Matrix C =

$$\begin{bmatrix} 0.0 & 0.544 & 0.213 & 0.384 & 0.102 & 0.544 & 0.213 & 0.384 & 0.102 \\ 0.875 & 0.0 & 0.0 & 0.0 & 0.0 & -0.372 & -0.146 & -0.263 & -0.070 \\ 0.0 & 0.447 & -0.108 & -0.360 & -0.100 & -0.447 & 0.108 & 0.360 & 0.100 \\ 0.0 & 0.264 & 0.261 & 0.106 & 0.0 & -0.264 & -0.261 & -0.106 & 0.0 \\ 0.0 & 0.0 & 0.144 & 0.588 & 0.175 & 0.0 & 0.144 & 0.588 & 0.175 \\ 0.0 & 0.0 & -0.291 & -0.132 & 0.0 & 0.0 & -0.291 & -0.132 & 0.0 \end{bmatrix}$$

The six rows of C are the six eigenvectors of the matrix A^T corresponding to its six dominant eigenvalues. Also here

$$F = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.497 & 3.522 & 0.0 & 0.0 \\ 0.0 & 0.0 & -3.522 & -0.497 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -1.297 & 2.513 \\ 0.0 & 0.0 & 0.0 & 0.0 & -2.513 & -1.297 \end{bmatrix} \qquad G = \begin{bmatrix} 1.281 & 1.281 \\ 0.0 & -0.875 \\ -1.248 & 1.248 \\ 0.0 & 0.0 \\ 2.188 & 2.188 \\ 0.0 & 0.0 \end{bmatrix}$$

The solution of the matrix Riccati equation for the reduced system is the 6x6 matrix

$$P = \begin{bmatrix} 2.790 & 1.109 & 0.173 & 0.145 & -0.991 & -0.150 \\ 1.109 & 3.247 & 0.506 & 0.425 & 0.0 & 0.0 \\ 0.173 & 0.506 & 0.980 & -0.269 & 0.0 & 0.0 \\ 0.145 & 0.425 & -0.269 & 1.499 & 0.0 & 0.0 \\ -0.991 & 0.0 & 0.0 & 0.0 & 0.761 & -0.294 \\ -0.150 & 0.0 & 0.0 & 0.0 & -0.294 & 0.173 \end{bmatrix}$$

Accordingly, K for this 6th order reduced model given by

$$K = R^{-1}\, G^{T}\, P$$

is computed; then the system matrix incorporating the sub-optimal feedback becomes

$$(A - B\,K\,C) = (A - B\,R^{-1}\,G^{T}\,P\,C)$$

The same is computed as

$$\begin{bmatrix} 0.0 & 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.270 & -0.050 & 6.0 & 0.0 & 3.270 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -3.333 & 3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\ -8.639 & -4.200 & -15.190 & -15.190 & -16.139 & -8.299 & -2.428 & -2.207 & -0.445 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 3.270 & 0.0 & 0.0 & 0.0 & -3.270 & -0.050 & 6.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.333 & 3.333 \\ 8.639 & -8.299 & -2.428 & -2.207 & -0.445 & -4.200 & -15.190 & -15.190 & -16.139 \end{bmatrix}$$

The response of the feedback system which is governed by the differential equation

$$\dot{\underline{x}} = (A - B\,K\,C)\,\underline{x}$$

is determined for a load disturbance of 0.01 p.u. of power in the first area.

3.8 COMPUTATION OF SUBOPTIMAL CONTROLLERS FOR 4TH ORDER AGGREGATION

Here the matrix C is computed as

$$\begin{bmatrix} 0.0 & 0.544 & 0.213 & 0.384 & 0.102 & 0.544 & 0.213 & 0.384 & 0.102 \\ 0.875 & 0.0 & 0.0 & 0.0 & 0.0 & -0.372 & -0.146 & -0.263 & -0.070 \\ 0.0 & 0.447 & -0.108 & -0.360 & -0.100 & -0.447 & 0.108 & 0.360 & 0.100 \\ 0.0 & 0.264 & 0.261 & 0.105 & 0.0 & -0.264 & -0.261 & -0.106 & 0.0 \end{bmatrix}$$

The four rows of C are the four eigenvectors of the matrix A^T corresponding to its four dominant eigenvalues.

$$F = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.497 & 3.522 \\ 0.0 & 0.0 & -3.522 & -0.497 \end{bmatrix} \qquad G = \begin{bmatrix} 1.281 & 1.281 \\ 0.0 & -0.875 \\ -1.248 & 1.248 \\ 0.0 & 0.0 \end{bmatrix}$$

The solution of the matrix Riccati equation for the reduced system is the 4x4 matrix

$$P = \begin{bmatrix} 0.836 & 1.109 & 0.173 & 0.145 \\ 1.109 & 3.247 & 0.506 & 0.425 \\ 0.173 & 0.506 & 0.980 & -0.269 \\ 0.145 & 0.425 & -0.269 & 1.499 \end{bmatrix}$$

Here also $K = R^{-1} G^T P$ is computed; the closed loop system matrix given by $(A - B K C) = (A - B R^{-1} G^T P C)$ is computed as

$$\begin{bmatrix} 0.0 & 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.270 & -0.050 & 6.0 & 0.0 & 3.270 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -3.333 & 3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\ -8.639 & -1.929 & -10.546 & -9.300 & -14.846 & -6.028 & 2.217 & 3.683 & 0.848 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 3.270 & 0.0 & 0.0 & 0.0 & -3.270 & -0.050 & 6.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.333 & 3.333 \\ 8.639 & -6.028 & 2.217 & 3.683 & 0.848 & -1.929 & -10.546 & -9.300 & -14.846 \end{bmatrix}$$

As in the case of 6th order aggregation, the response of the closed loop system governed by the differential equation

$$\dot{\underline{x}} = (A - B\,K\,C)\,\underline{x}$$

is determined for a load disturbance of 0.01 p.u. of power in the first area. The optimal closed loop response obtained previously[3] for the original 9th order system is used for comparison.

The optimal and suboptimal responses for the cases of 6th and 4th order aggregations for each of the state variables of interest are drawn on the same figure to give an idea of the nearness of the suboptimal response to that of the optimal one. Figures 3.1, 3.2 and 3.3 are drawn for the state variables viz. frequency deviation Δf_1 in Area 1, frequency deviation Δf_2 in Area 2 and tieline power error $\Delta P_{tie\,1}$ respectively.

3.9 COMPUTATION OF DEGRADATION IN PERFORMANCE

The trace of the matrix T* of (3.18) obtained for optimal control is computed as 8.8053. For the suboptimal control by 6th order aggregation, the trace of the matrix T given by (3.30) is computed as 8.9189. Similarly for the 4th order aggregation the trace of T is computed as 9.5608.

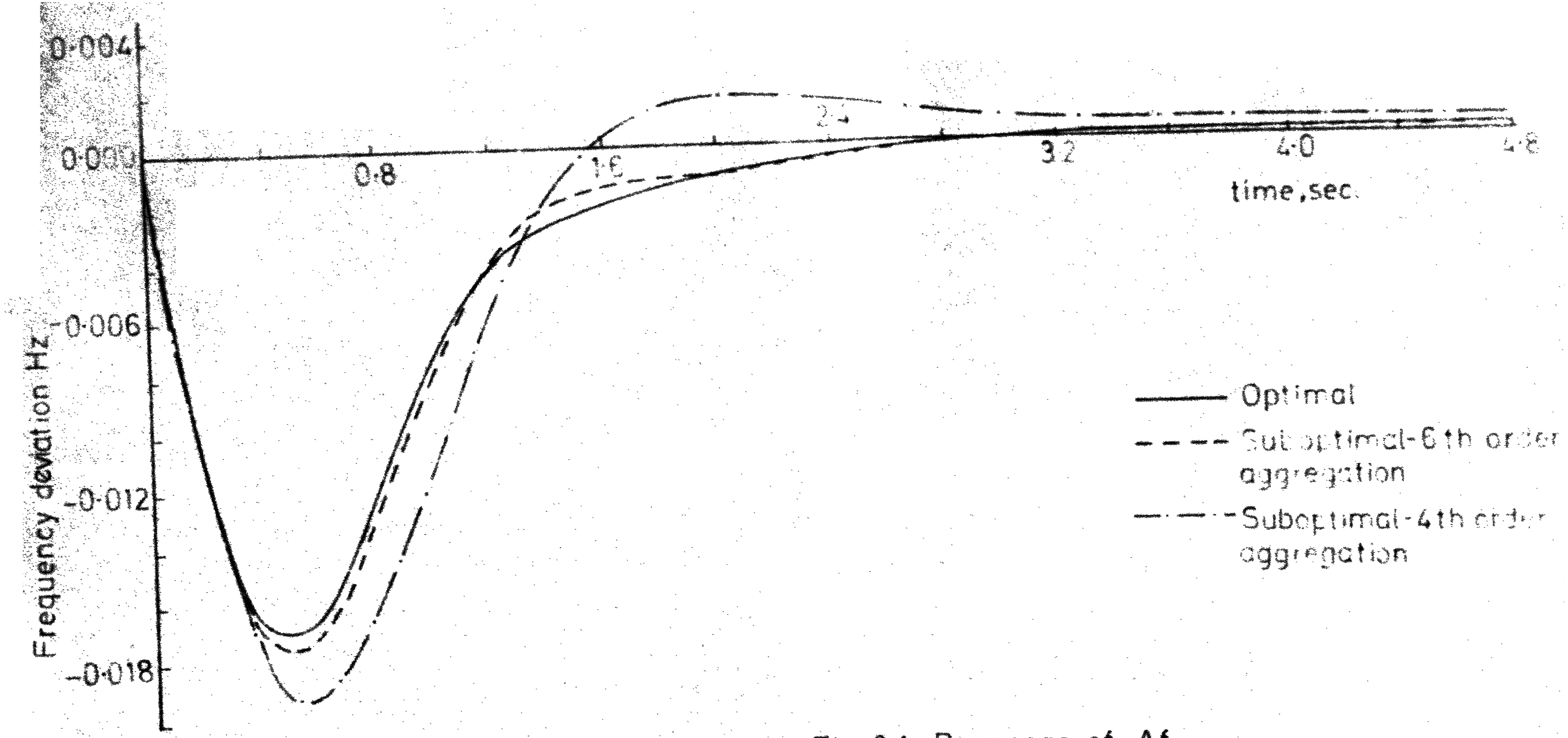


Fig. 3·1 Response of Δf_1

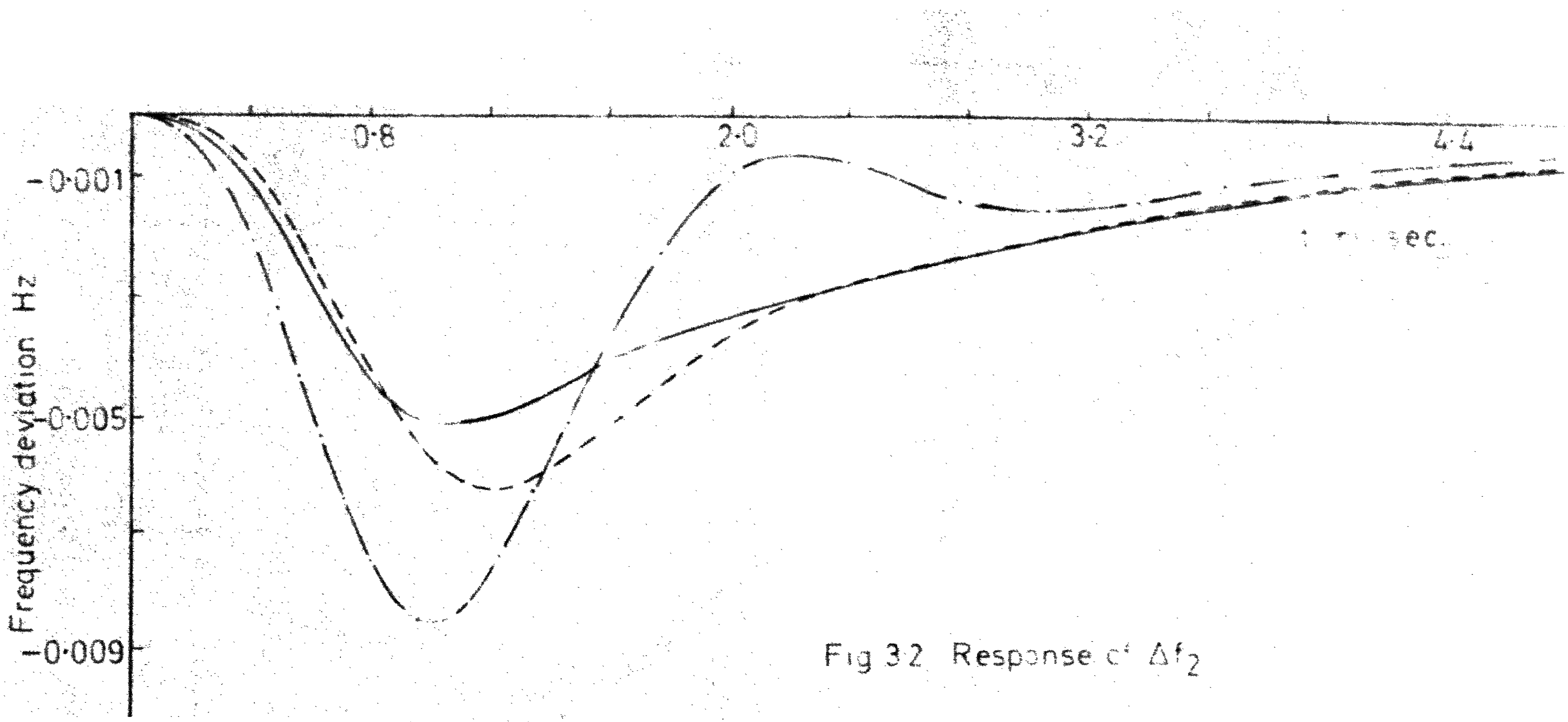


Fig 3·2 Response of Δf_2

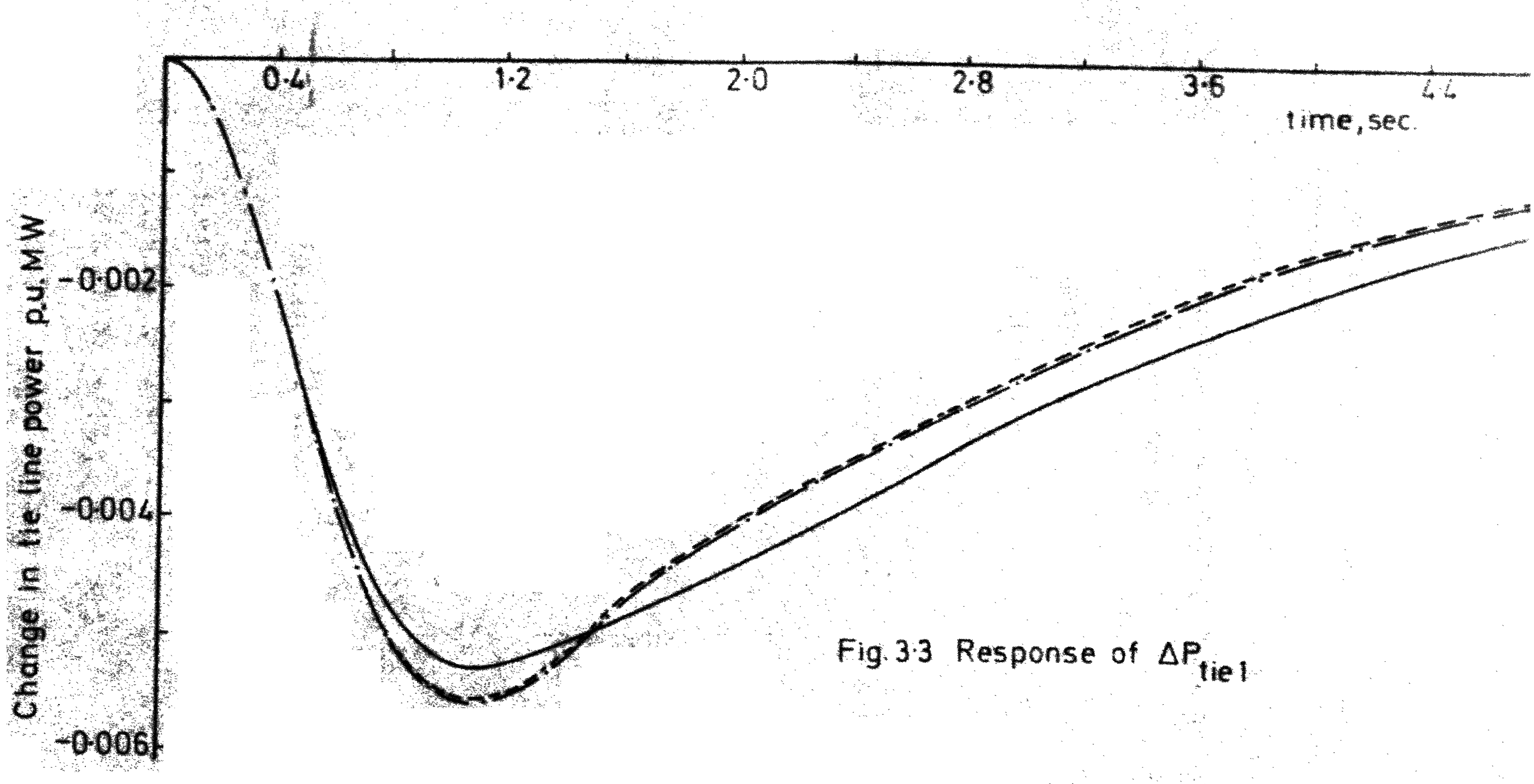


Fig. 3·3 Response of ΔP_{tie1}

3.10 STABILITY OF THE CLOSED LOOP SYSTEM - COMPUTATION OF EIGENVALUES

The eigenvalues (some of which are complex) of the closed loop system matrix (A - B K C) in the case of 6th order aggregation are computed as

1) -0.505024795 + j0.0
2) -0.932741128 + j0.0
3) -1.623612400 + j0.0
4) -1.842025460 + j3.731263460
5) -1.842025460 - j3.731263460
6) -2.872359200 + j3.451757180
7) -2.872359200 - j3.451757180
8) -13.265108300 + j0.0
9) -13.290158100 + j0.0

In the case of 4th order aggregation the closed loop eigenvalues are:

1) -0.505024795 + j0.0
2) -1.296577550 + j2.512660310
3) -1.296577550 - j2.512660310
4) -1.497734720 + j0.0
5) -1.623612400 + j0.0
6) -1.842025460 + j3.731263460
7) -1.842025460 - j3.731263460
8) -13.265108300 + j0.0
9) -13.290158100 + j0.0

The eigenvalues of the open loop system matrix A of (3.1) are also given below for comparison:

1)	0.0	+ j0.0
2)	0.0	+ j0.0
3)	-0.497296223	+ j3.522088710
4)	-0.497296223	- j3.522088710
5)	-1.296577540	+ j2.512660280
6)	-1.296577540	- j2.512660280
7)	-1.623612400	+ j0.0
8)	-13.265108300	+ j0.0
9)	-13.290158100	+ j0.0

3.11 CONCLUSIONS

In this Chapter, the theory of aggregation due to Aoki is applied to reduce the model and compute suboptimal controllers for the two-area 9th order LFC system. Two cases of model reduction by aggregation are considered, 6th order and 4th order. The suboptimal control by 6th order aggregation is accurate for all practical purposes as could be seen from Figures 3.1, 3.2 and 3.3. This fact is confirmed by a) the increase in the performance index figure which is only 1.3% of the optimal one and b) the closed loop eigenvalues which are more stable than those obtained by 4th order aggregation. In the case of 4th order, the error in response is considerable; also the increase in the performance index figure is 8.6% of the optimal one.

The suboptimal control by aggregation considerably reduces the computational effort and time. This could be seen from the fact that the matrix Riccati equation of order 45 would have to be solved if optimal control were to be determined, whereas in the case of 6th order aggregation, the order of matrix Riccati equation to be solved is only 21. If the results obtained with 4th order can be considered to be tolerable for all practical purposes, then the order of matrix Riccati equation to be solved is only 10.

In the above method correct knowledge of the eigenvectors of the original system is essential. However, Aoki[6] has given a method for weakly coupled systems which requires only an approximate knowledge of the eigenvectors. This method involves an iterative process for computing suboptimal control.

The above results show that the accuracy of representation increases with the order of the aggregated model. Thus a tradeoff has to be made between accuracy of representation and saving in computational effort and the system reduced to the appropriate order.

CHAPTER 4

SUBOPTIMAL REGULATION OF A NONLINEAR LOAD FREQUENCY CONTROL SYSTEM

In LFC systems the tieline power is a function of the sine of the angular difference of the voltage vectors at either end of the tieline. Under small load disturbances this tieline power becomes a function of the angular difference itself and the system can be treated as linear. However, under large load disturbances it has to be treated as nonlinear having the sine term as the nonlinearity. Such LFC systems will henceforth be termed as nonlinear LFC systems having tieline power nonlinearity.

In Chapters 2 and 3 methods are presented for the optimal and suboptimal regulation of an LFC system. The analysis therein is based on the assumption that load disturbances are small and thus linear analysis and design methods are justified. In practical systems, load disturbances are not necessarily confined to be small; and any analysis made should also cover large disturbances. As explained above, the tieline power nonlinearity is an important nonlinearity to be considered under such large disturbances.

Whereas well defined, closed form methods are available for the optimal and suboptimal regulation of linear dynamical systems, such closed form solutions have not been dealt with in literature for the regulation of nonlinear dynamical systems. However, Lukes[7] has given a method for the optimal regulation of autonomous nonlinear dynamical systems which have a linear term in control. The hypothesis used therein is that with the linear part of the system and the linear control term, a stable closed loop system can be constructed by employing linear optimization methods. As LFC systems fit into the above class of nonlinear dynamical systems, Lukes' method is applicable to the same. Section 4.2 describes Lukes' method in brief along with statements of some important theorems which contain the main results.

Lukes' method suffers from the disadvantage that the amount of computation increases considerably, as the order of the system goes up. Only systems of small order (say 6 or less) can be treated without any computational difficulties because apart from the usual solution of matrix Riccati equation, the problem reduces to the solution of a system of linear algebraic equations and the order of the system of equations increases steeply as the order of the system goes up. This is explained

further in Section 4.3. Hence it is imperative that some method of reduction be adopted for first reducing the model of the multiarea LFC system before Lukes' method is applied to the same. Section 4.4 gives a method of reduction of the nonlinear LFC systems, taking advantage of the sine term in the tieline power nonlinearity occurring therein. In Section 4.5 the method is applied to a 4th order example which could be considered as a single area LFC system tied to an infinite system. Two types of study are made on this 4th order example: (i) Lukes' method for determination of nonlinear regulation is applied to the system as such; (ii) the system is reduced to a 3rd order nonlinear system; the optimal controller determined by Lukes' method for this reduced system is used as a suboptimal controller for the original system. Thus this example serves to evaluate the closeness of the suboptimal response to the optimal one as well as the degradation in performance using the suboptimal controller with respect to the optimal performance.

4.2 OPTIMAL REGULATION OF NONLINEAR DYNAMICAL SYSTEMS

A. Formulation of the Problem:

The problem is formulated in terms of a control system equation in R^n,

$$\dot{\underline{x}} = F(\underline{x}, \underline{u}) \tag{4.1}$$

and a performance integral

$$J = \int_0^{\infty} G(\underline{x}, \underline{u})\, dt \tag{4.2}$$

We seek an r-dimensional vector feedback control function of the state $\underline{x}$, $\underline{u} = \underline{u}(\underline{x})$, which minimizes the integral in (4.2) subject to the constraint given in (4.1). Equation (4.1) is an autonomous differential equation where $F(\underline{x},\underline{u})$ and $G(\underline{x},\underline{u})$ can be represented in the form

$$F(\underline{x}, \underline{u}) = A\,\underline{x} + B\,\underline{u} + \underline{f}(\underline{x}, \underline{u}) \qquad (4.3)$$

$$G(\underline{x}, \underline{u}) = \underline{x}.Q\,\underline{x} + 2\underline{x}.N\,\underline{u} + \underline{u}.R\,\underline{u} + g(\underline{x},\underline{u}) \qquad (4.4)$$

Here the inner product notation

$$\underline{x}.\underline{y} = \sum_{k=1}^{n} x_k\, y_k$$

and the partial differential notation $f_u(\underline{x},\underline{u}) = \dfrac{\partial \underline{f}(\underline{x},\underline{u}}{\partial \underline{u}}$ are used.

B. Definitions:

(i) A real matrix is called a stability matrix if all of its eigenvalues have negative real parts.

(ii) The pair of matrices (A,B) as well as the control system defined by $F(\underline{x},\underline{u})$ in (4.3) is stabilizable if there exists a real matrix for which (A + B K) is a stability matrix.

This is explained as follows. We consider the class of feedback controls which are of the form

$$\underline{u} = \underline{u}(\underline{x}) = K\,\underline{x} + \underline{h}(\underline{x}) \qquad (4.5)$$

where $\underline{h}(\underline{x})$ denotes the higher order terms. The real matrices K are always selected so that $\underline{u}(\underline{x})$ stabilizes the system

described in (4.1); that is, we demand that in

$$\underline{F}(\underline{x},\underline{u}(\underline{x})) = (A + B\ K)\ \underline{x} + B\ \underline{h}(\underline{x}) + \underline{f}(\underline{x},\underline{u}(\underline{x})) \quad (4.6)$$

(A + B K) should be a stability matrix.

(iii) A C^w stabilizing feedback control $\underline{u}_*(\underline{x}) = K_*\ \underline{x}_* + \underline{h}_*(\underline{x})$ is optimal for the process in (4.1) with respect to the performance integral given in (4.2) if for every stabilizing feedback control $\underline{u}(\underline{x}) = K\ \underline{x} + \underline{h}(\underline{x})$, there exists a neighbourhood N_u of the origin in which

$$J(\underline{x}_o,\ \underline{u}_*) \leq J(\underline{x}_o,\underline{u}) \quad (4.7)$$

C. Basic Assumptions:

(i) It is assumed that $F(\underline{x},\underline{u})$ and $G(\underline{x},\underline{u})$ are defined in some neighbourhood of the origin in R^{n+r} and can be represented in the form of (4.3) and (4.4) respectively.

(ii) In (4.4) we assume

$$\begin{bmatrix} Q & N \\ N^T & B \end{bmatrix} > 0$$

(iii) The fundamental hypothesis is that $F(\underline{x},\underline{u})$ is stabilizable.

(iv) It is assumed that $F(\underline{x},\underline{u})$ and $G(\underline{x},\underline{u})$ are real analytic about the origin in R^{n+r}. By this we understand the terms $\underline{f}(\underline{x},\underline{u})$ and $\underline{g}(\underline{x},\underline{u})$ in (4.3) and (4.4) to be real convergent power series about the origin beginning with second and higher order terms in $(\underline{x},\underline{u})$ respectively. In

this situation we admit every $\underline{h}(\underline{x})$ in (4.5) given by real power series converging about the origin and beginning with second order terms. The feedback controls are called C^W stabilizing controls.

D. Main Theorem and Theorem Concerning Truncated System

The main results are given in the main theorem below:

Main Theorem:

For the C^W stabilizable control process in R^n

$$\dot{\underline{x}} = F(\underline{x},\underline{u}) = A\,\underline{x} + B\,\underline{u} + \underline{f}(\underline{x},\underline{u}) \tag{4.8}$$

with the performance integral

$$J(\underline{x}_0,\underline{u}) = \int_0^\infty G(\underline{x},\underline{u})dt$$

$$= \int_0^\infty \left[\begin{pmatrix} \underline{x} \\ \underline{u} \end{pmatrix} \cdot \begin{pmatrix} Q & N \\ N^T & R \end{pmatrix} \begin{pmatrix} \underline{x} \\ \underline{u} \end{pmatrix} + \underline{g}(\underline{x},\underline{u}) \right] dt \quad \text{.. (4.9)}$$

there exists an optimal C^W stabilizing feedback control $\underline{u}_*$. The optimal control solves the functional equation:

$$F_u(\underline{x}_0,\ \underline{u}_*(\underline{x}_0)).J_x(\underline{x}_0,\underline{u}_*) + G_u(\underline{x}_0,\underline{u}_*(\underline{x}_0)) = 0 \tag{4.10}$$

for all $\underline{x}_0$ near the origin and is unique in that

(i) $\underline{u}_*$ is the unique C^W solution to (4.10).

(ii) $\underline{u}_*$ is the unique C^W stabilizing feedback control.

(iii) $\underline{u}_*$ synthesizes the unique optimal open-loop control.

Furthermore, $\underline{u}_*(\underline{x}) = K_* \, \underline{x} + \underline{h}_*(\underline{x})$ and $J(\underline{x}_o, \underline{u}_*) = \underline{x}_o . P \, \underline{x}_o + j_*(\underline{x}_o)$, where the lowest order terms are given by matrices K_* and $P_* > 0$ depending upon only A, B, Q, R and N.

The truncated or linear part of the dynamic equation as well as the quadratic part of the performance integral are then formulated into a linear control problem. The following is the statement of the theorem concerning the truncated system so formulated:

<u>Theorem concerning the Truncated System</u>:

For the special case of the Main Theorem in which

$$\dot{\underline{x}} = A \, \underline{x} + B \, \underline{u} \tag{4.11}$$

and

$$J(\underline{x}_o, \underline{u}) = \int_o^\infty \begin{pmatrix} \underline{x} \\ \underline{u} \end{pmatrix} . \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{pmatrix} \underline{x} \\ \underline{u} \end{pmatrix} dt \tag{4.12}$$

the optimal control is $\underline{u}_*(\underline{x}) = K_* \, \underline{x}$, where $K_* = -R^{-1}\left[N^T + B^T P_*\right]$.
Here $P_* > \underline{0}$ solves the matrix equation

$$(Q - NR^{-1}N^T) + P_*(A - BR^{-1}N^T) + (A - BR^{-1}N^T)P_* - P_*(BR^{-1}B^T)P_* = 0 \tag{4.13}$$

and is the unique positive definite solution.

$K_* \, \underline{x}$ is a global optimal control; finally

$$J(\underline{x}_o, \underline{u}_*) = \underline{x}_o . P_* \, \underline{x}_o \tag{4.14}$$

The theorem for the truncated system is first proved with the help of intermediate Lemmas[7]. The proof is then extended to the Main Theorem.

E. Calculation of Power Series for $\underline{u}_*(\underline{x})$ and $J(\underline{x},\underline{u}_*)$

Since $\underline{u}_*(\underline{x})$, $J_*(\underline{x}) = J(\underline{x},\underline{u}_*)$ and $F_*(\underline{x}) = F(\underline{x},\underline{u}_*(\underline{x}))$ are analytic about the origin they can be expanded in power series as:

$$\underline{u}_*(\underline{x}) = \underline{u}_*^{(1)}(\underline{x}) + \underline{u}_*^{(2)}(\underline{x}) + \ldots \tag{4.15}$$

$$J_*(\underline{x}) = J^{(2)}(\underline{x}) + J^{(3)}(\underline{x}) + \ldots \tag{4.16}$$

$$F_*(\underline{x}) = F^{(1)}(\underline{x}) + F^{(2)}(\underline{x}) + \ldots \tag{4.17}$$

The lowest order terms in the above series have been computed as solutions to the truncated problem.

$$\underline{u}_*^{(1)}(\underline{x}) = K_* \ \underline{x} \tag{4.18}$$

$$J^{(2)}(\underline{x}) = \underline{x}.P_*\underline{x} \tag{4.19}$$

$$F^{(1)}(\underline{x}) = A_* \ \underline{x} \tag{4.20}$$

where $\quad A_* = (A + B\ K_*)$ (4.21)

The computation in the case of truncated system reduces to solving the quadratic matrix equation

$$(Q-NR^{-1}N^T) + P(A-BR^{-1}N^T) + (A-BR^{-1}N^T)P - P(BR^{-1}B^T)P = 0 \quad \ldots (4.22)$$

for $P = P_* > 0$ and computing K_* and A_* by the formulas

$$K_* = -R^{-1}(N^T + B^T\ P_*) \tag{4.23}$$

and that given in (4.21).

The following procedure is followed for computing the remaining terms in the power series.

In one of the Lemmas given in Lukes theory the following relation is proved:

$$F(\underline{x},\underline{u}_*(\underline{x})).J_x(\underline{x},\underline{u}_*) + G(\underline{x},\underline{u}_*(\underline{x})) = 0 \qquad (4.24)$$

about the origin.

Also from the main theorem we have:

$$F_u(\underline{x},\underline{u}_*(\underline{x})).J_x(\underline{x},\underline{u}_*) + G_u(\underline{x},\underline{u}_*(\underline{x})) = 0 \qquad (4.25)$$

After substituting for $F(\underline{x},\underline{u}_*(\underline{x}))$, $G(\underline{x},\underline{u}_*(\underline{x}))$ and $F_u(\underline{x},\underline{u}_*(\underline{x}))$, $G_u(\underline{x},\underline{u}_*(\underline{x}))$, these equations can be written in the form:

$$\left[A_*\underline{x} + B(\underline{u}_* - K_*\underline{x}) + \underline{f}(\underline{x},\underline{u}_*)\right] . J_x(\underline{x},\underline{u}_*)$$
$$+ \underline{x}.Q\underline{x} + 2\underline{x}.N\underline{u}_* + \underline{u}_*.R\,\underline{u}_* + \underline{g}(\underline{x},\underline{u}_*) = 0 \qquad (4.26)$$

$$\underline{u}_*(\underline{x}) = -\frac{1}{2}R^{-1}\left[(B + \underline{f}_u)^T J_x(\underline{x},\underline{u}_*) + 2N^T\underline{x} + \underline{g}_u\right] \qquad \ldots (4.27)$$

Hence

$$A_*\underline{x}.J_x(\underline{x},\underline{u}_*) = -\left[B(\underline{u}_*-K_*\underline{x}) + \underline{f}(\underline{x},\underline{u}_*)\right].J_x(\underline{x},\underline{u}_*)$$
$$- 2\underline{x}.N\underline{u}_* - \underline{u}_*.R\underline{u}_* - \underline{g}(\underline{x},\underline{u}_*) - \underline{x}.Q\underline{x} \qquad \ldots (4.28)$$

$$\underline{u}_*(\underline{x}) = -\frac{1}{2}R^{-1}\left[(B + \underline{f}_u)^T . J_x(\underline{x},\underline{u}_*) + 2N^T\underline{x}+\underline{g}_u(\underline{x},\underline{u}_*)\right] \qquad (4.29)$$

Substituting the power series for $\underline{u}_*$ and $J(\underline{x},\underline{u}_*)$ and then selecting the mth order terms from the former equation and the lth order terms from the latter, and using the equality $-\left[B^T J_x^{(2)}(\underline{x}) + 2N^T\underline{x} + 2R\,\underline{u}^{(1)}\right] = 0$ in (4.28) the following equations are obtained.

$$A_* \underline{x}.J_x^{(m)}(\underline{x}) = -\sum_{l=3}^{m-1} B\underline{u}_*^{(m-l+1)}(\underline{x}).\ J_x^{(l)}(\underline{x})$$

$$-\sum_{l=2}^{m-1} \underline{f}^{(m-l+1)}(\underline{x},\underline{u}_*)\ .\ J_x^{(l)}(\underline{x})$$

$$-2\sum_{l=2}^{(m-1)/2} \underline{u}_*^{(l)}(\underline{x})\ .\ R\underline{u}_*^{(m-l)}(\underline{x})$$

$$-\underline{u}_*^{(m/2)}(\underline{x}).R\underline{u}_*^{(m/2)}(\underline{x}) - g^{(m)}(\underline{x},\underline{u}_*) \quad (4.30)$$

$$\text{for } m = 3,4,5\ldots$$

$$\underline{u}_*^{(l)}(\underline{x}) = -\frac{1}{2}R^{-1}\Big[B^T J_x^{(l+1)}(\underline{x}) + \sum_{j=1}^{l-1}(\underline{f}_u)^{T(j)} J_x^{(l-j+1)}(\underline{x})$$

$$+ \underline{g}_u^{(l)}(\underline{x},\underline{u}_*)\Big] \quad \text{for } l = 2,3,\ldots \qquad (4.31)$$

Here $[l]$ denotes the integer part of l and the term with $u^{(m/2)}$ in it is to be omitted for m odd.

With the help of the above two equations and starting with $\underline{u}_*^{(1)}(\underline{x}) = K_*\underline{x}$ and $J^{(2)}(\underline{x}) = \underline{x}.P_*\underline{x}$ the terms in the following sequence can be computed consecutively:

$$J^{(2)}(\underline{x}),\ \underline{u}_*^{(1)}(\underline{x}),\ J^{(3)}(\underline{x}),\ \underline{u}_*^{(2)}(\underline{x}),\ J^{(4)}(\underline{x}),\underline{u}_*^{(3)}(\underline{x}),\ldots$$

thereby generating the power series for $J_*(\underline{x})$ and $\underline{u}_*(\underline{x})$. The computation of successively higher order terms reduces to solving successively higher order systems of linear algebraic equations.

F. Extension of Theorems to the Differentiable Case

With some additional assumptions on $\underline{f}(\underline{x},\underline{u})$, $\underline{g}(\underline{x},\underline{u})$ and $\underline{h}(\underline{x})$ the proofs of the above theorems are extended to the case where $F(\underline{x},\underline{u})$ and $G(\underline{x},\underline{u})$ are twice continuously differentiable

about the origin in R^{n+r} and $\underline{h}(\underline{x})$ is at least once continuously differentiable about the origin.

4.3 REDUCTION OF MULTIVARIABLE DYNAMICAL SYSTEMS

In Section 4.2 it is stated that the optimal feedback control for a nonlinear system is of a series form and the computation of successively higher order terms in the series ultimately reduces to solving successively higher order systems of linear algebraic equations. For all practical purposes it is sufficient to stop with $J^{(3)}(\underline{x})$ and $\underline{u}_*^{(2)}(\underline{x})$ because the effect of the higher terms is negligible. Hence the problem further reduces to that of assuming a third degree form for $J^{(3)}(\underline{x})$ and solving for the coefficients of the same. Section 4.5 gives the application of the method to a 4th order nonlinear system and thus provides an insight into the method. The order of the linear system of algebraic equations that is to be solved for determining $J^{(3)}(\underline{x})$, in the case of systems of order 2 to 8 is given in Table 4.1.

Table 4.1

Order of system vs. order of simultaneous equations

Order of the system	Order of simultaneous equations to be solved
2	4
3	10
4	20
5	35
6	56
7	84
8	120

Table 4.1 shows that the order of linear algebraic equations that is to be solved increases steeply as the order of the system goes up. Further the smallest multiarea LFC system, viz. a two-area system is consisting of 8 state variables, this order increasing to much more than 8 in three-area and higher order multi-area systems. Hence considerable amount of computational difficulty can be expected if Lukes' method for nonlinear regulation is applied to the original model of the multi-area system. This suggests that if the order can be reduced by some means, the nonlinear regulation method provides us with a very good suboptimal control for the original system in the shape of optimal control of the reduced system.

Fortunately, in the case of LFC systems, the tieline power nonlinearity is suited for such model reduction. For this purpose, the method of aggregation due to Aoki[6] is suitably modified and applied (Chapter 3). This is dealt with in Section 4.4 as applied to a two-area LFC system.

4.4 REDUCTION OF MULTIAREA LFC SYSTEMS AND REGULATION

The dynamics of the two-area nonlinear LFC system, which is linear in the control $\underline{u}$, is given by

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \underline{f}(\underline{x}) \tag{4.32}$$

where $\underline{x}$ is an 8-vector and $\underline{u}$ is a 2-vector.

The description of the matrices A, B and also that of the nonlinear part $\underline{f}(\underline{x})$ in (4.32) are given in Chapter 5.

The performance index to be minimized is taken as

$$J = \int_0^{\infty} (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u})\, dt \qquad (4.33)$$

It is required to reduce the 8th order system described above to a 5th order model. Recalling the theory of aggregation presented in Chapter 3, the transformation that is made use of, for this purpose, is

$$\underline{z} = C \underline{x} \qquad (4.35)$$

where $\underline{z}$ is a 5-element vector and C is a 5x8 matrix. The following method extends the technique of aggregation to compute C in the case of nonlinear LFC systems. For this the special type of vector function $\underline{f}(\underline{x})$ occurring therein is manipulated. $\underline{f}(\underline{x})$ in the two area LFC system is of the form

$$\underline{f}(\underline{x}) = \begin{bmatrix} 0 \\ k_1 \mathrm{Sin}2\pi\,(x_1-x_5) + k_2\{\mathrm{Cos}2\pi(x_1-x_5)-1\} \\ 0 \\ 0 \\ 0 \\ k_3 \mathrm{Sin}2\pi\,(x_1-x_5) + k_4\,\{\mathrm{Cos}2\pi(x_1-x_5)-1\} \\ 0 \\ 0 \end{bmatrix} \qquad (4.35)$$

The second and sixth nonzero elements of (4.35) can also be written as

$$f_2(\underline{x}) = \left[\frac{k_1 \mathrm{Sin}2\pi(x_1-x_5) + k_2\{\mathrm{Cos}2\pi(x_1-x_5)-1\}}{2\pi\,(x_1-x_5)}\right] 2\pi(x_1-x_5)$$

$$f_6(\underline{x}) = \left[\frac{k_3 \mathrm{Sin}2\pi(x_1-x_5) + k_4\,\{\mathrm{Cos}2\pi(x_1-x_5)-1\}}{2\pi\,(x_1-x_5)}\right] 2\pi\,(x_1-x_5)$$

$$\ldots(4.36)$$

Let

$$\theta_1(\underline{x}) = \left[\frac{k_1 \operatorname{Sin} 2\pi(x_1-x_5) + k_2 \{\operatorname{Cos} 2\pi(x_1-x_5) - 1\}}{2\pi(x_1-x_5)}\right] \qquad (4.37)$$

and

$$\theta_2(\underline{x}) = \left[\frac{k_3 \operatorname{Sin} 2\pi(x_1-x_5) + k_4 \{\operatorname{Cos} 2\pi(x_1-x_5) - 1\}}{2\pi(x_1-x_5)}\right] \qquad (4.38)$$

These scalar functions $\theta_1(\underline{x})$ and $\theta_2(\underline{x})$ are plotted versus $2\pi(x_1-x_5)$ or $\Delta\delta_{12}$ in the usual range, say 0 to $\pi/2$ rad. A typical curve obtained for $\theta_1(\underline{x})$ with k_1 and k_2 taken as 0.866 and 0.5 respectively, is plotted in Figure 4.1. It is seen that the variation of $\theta_1(\underline{x})$ is nearly linear in the range taken. The plot up to a lesser angle will ofcourse be more approximately linear. Let the maximum excursion of $\Delta\delta_{12}$ be σ ($< \frac{\pi}{2}$ radians). For the purpose of computing C the value of $\theta_1(\underline{x})$ read off from the plot given in Figure 4.1 at an angle midway, i.e. $\frac{\sigma}{2}$ radians is taken to be constant for the whole range from 0 to σ radians. Let the values read off at $\frac{\sigma}{2}$ radians for $\theta_1(\underline{x})$ and $\theta_2(\underline{x})$ be h_1 and h_2 respectively. Thus the second and sixth nonzero elements of $\underline{f}(\underline{x})$ given in (4.36) can be written as

$$\begin{aligned} f_2(\underline{x}) &= h_1 \; 2\pi(x_1-x_5) \\ f_6(\underline{x}) &= h_2 \; 2\pi(x_1-x_5) \end{aligned} \qquad (4.39)$$

Substituting this linear expression in the place of $\underline{f}(\underline{x})$ in (4.32) and then incorporating the corresponding terms in the matrix A, (4.32) can be written as:

$$\dot{\underline{x}} = H\,\underline{x} + B\,\underline{u} \qquad (4.40)$$

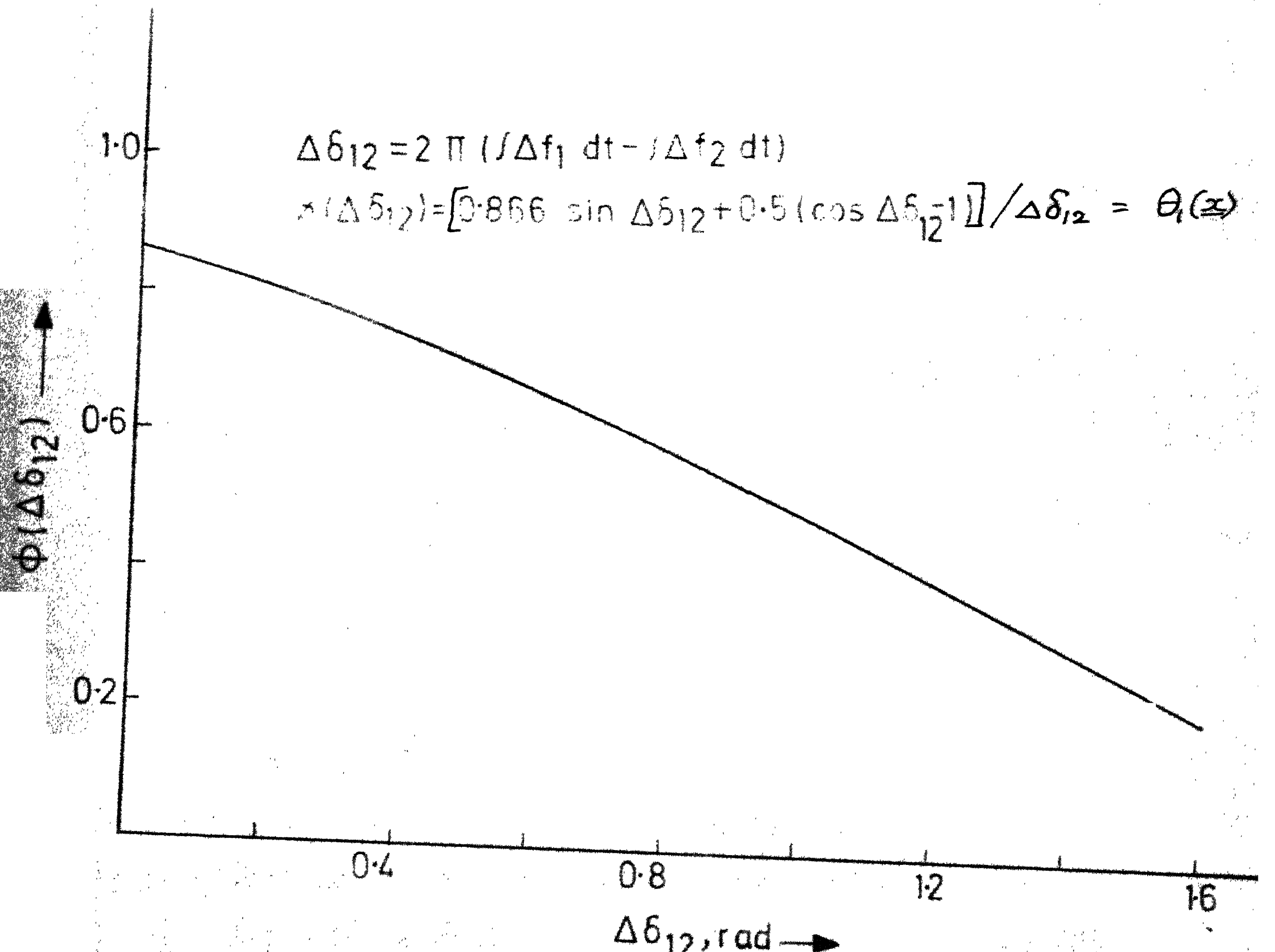


Fig. 4·1 Plot of ϕ $(\Delta\delta_{12})$ Vs $\Delta\delta_{12}$

The matrix H of (4.40) is utilized for the purpose of computation of the matrix C using the theory of aggregation. Thus the five rows of C consist of the eigenvectors of the matrix H^T corresponding to the five dominant eigenvalues. Reverting back to (4.32) and multiplying the same through out by C

$$C\,\dot{\underline{x}} = C\,A\,\underline{x} + C\,B\,\underline{u} + C\,\underline{f}(\underline{x}) \tag{4.41}$$

Denoting

$$G = C\,B \tag{4.42}$$

and noting that

$$\dot{\underline{z}} = C\,\dot{\underline{x}} \tag{4.43}$$

equation (4.40) can be written as

$$\dot{\underline{z}} = C\,A\,\underline{x} + G\,\underline{u} + C\,\underline{f}(\underline{x}) \tag{4.44}$$

Using the generalized inverse concept, $\underline{x}$ in (4.34) can be written as

$$\underline{x} = C^T(C\,C^T)^{-1}\,\underline{z} \tag{4.45}$$

Thus $C\,A\,\underline{x}$ in (4.44) becomes

$$C\,A\,\underline{x} = C\,A\,C^T(C\,C^T)^{-1}\,\underline{z} \tag{4.46}$$

Denoting

$$\bar{F} = C\,A\,C^T(C\,C^T)^{-1} \tag{4.47}$$

and using the relation (4.45) to express the terms in $\underline{x}$ of (4.44) in terms of $\underline{z}$, (4.44) can be written as

$$\dot{\underline{z}} = \bar{F}\,\underline{z} + G\,\underline{u} + \bar{g}(\underline{z}) \tag{4.48}$$

where $\bar{g}(\underline{z}) = C\,\underline{f}\left[C^T(C\,C^T)^{-1}\underline{z}\right]\cancel{z}$ (4.49)

and is a five element vector. The linear terms of $\underline{z}$ in the

vector function $\overline{g}(\underline{z})$ of (4.48) are incorporated into the matrix $\overline{F}$ and (4.48) is finally written as

$$\dot{\underline{z}} = F\,\underline{z} + G\,\underline{u} + \underline{g}(\underline{z}) \tag{4.50}$$

Here F is a 5x5 matrix, G is a 5x2 matrix and $\underline{g}(\underline{z})$ is a 5-element vector function of $\underline{z}$. Equation (4.50) describes the dynamics of the reduced (5th order) system. The integral performance index for the reduced system is taken as

$$J_M = \int_0^{\infty} (\underline{z}^T Q_M\,\underline{z} + \underline{u}^T R\,\underline{u})\,dt \tag{4.51}$$

where

$$Q_M = (C\,C^T)^{-1}\,C\,Q\,C^T(C\,C^T)^{-1} \tag{4.52}$$

and

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.53}$$

Lukes' method is now applied for the regulation of the nonlinear 5th order control problem described by (4.50) and (4.51). Let the optimal control vector $\underline{u}$ thus determined for the 5th order system be given by

$$\underline{u}_*(\underline{z}) = \hat{K}_1\,\underline{z} + \hat{K}_2(\underline{z}) + \hat{K}_3(\underline{z}) + \ldots \tag{4.54}$$

where $\hat{K}_1$ is a 2x5 constant matrix, $\hat{K}_2(\underline{z})$ is a vector function consisting of only second degree terms in $\underline{z}$ etc. Using the transformation $\underline{z} = C\,\underline{x}$ and writing (4.54) in terms of $\underline{x}$ only, the resulting expression, say $\underline{v}_*(\underline{x})$ is written as

$$\underline{v}_*(\underline{x}) = K_1\,\underline{x} + K_2(\underline{x}) + K_3(\underline{x}) + \ldots \tag{4.55}$$

where $K_1 = \hat{K}_1 C$ is a 2x8 constant matrix and $K_2(\underline{x})$, $K_3(\underline{x})$ are second and third order power series in $\underline{x}$. Thus the control described in (4.55) becomes the suboptimal feedback control for the original system.

In Section 4.5 below, the above theory is applied to a 4th order system which could be considered as a single-area LFC system tied to an infinite system. In Chapter 5 computations are presented for reduction of the two-area LFC system of 8th order first to a 5th order model and then to a 3rd order model. Suboptimal controls for the original system are determined in both these cases.

4.5 EXAMPLE OF A 4TH ORDER SYSTEM

In this section, optimal feedback control is first determined for the 4th order nonlinear system described below by directly applying Lukes' method to the same. Next, it is reduced to a 3rd order nonlinear model by the method presented in Section 4.4. The optimal control constructed for the 3rd order model is then used as a suboptimal control for the 4th order system. The closeness of the suboptimal response to that of the optimal response is examined; also the degradation in performance with suboptimal control, is evaluated.

A. Nonlinear Regulation of a 4th Order System

The dynamics of the 4th order nonlinear system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 \\ 0 & 0 & -\frac{1}{T_{t1}} & \frac{1}{T_{t1}} \\ 0 & -\frac{1}{R_1 T_{gv1}} & 0 & -\frac{1}{T_{gv1}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -\frac{f^*T_{1p}}{2H_1} [0.5(\cos 2\pi x_1 - 1.0) + 0.866 \operatorname{Sin} 2\pi x_1] \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{gv1}} \end{bmatrix} u \qquad .. \quad (4.56)$$

As stated before, this 4th order system could be interpreted as a single area LFC system connected to an infinite system. Accordingly the state variables x_1 to x_4 respectively represent $\int \Delta f dt$, Δf, ΔP_g and ΔX_{gv}. The notation as well as the parameters used here are the same as in Appendix A except for T_{1p} which is equal to 0.1.

Equation (4.56) is written in the vector form as

$$\underline{\dot{x}} = A\,\underline{x} + \underline{f}(\underline{x}) + B\,\underline{u} \qquad (4.57)$$

In (4.56) only the differential equation pertaining to x_2 is having the nonlinearity. The only nonzero element of $\underline{f}(\underline{x})$ in (4.57) can be approximated by expanding $\operatorname{Sin} 2\pi x_1$

and $\mathrm{Cos}\,2\pi x_1$ up to second order terms as

$$-\frac{f^*T_{1p}}{2H_1}\left[-0.5\frac{(2\pi x_1)^2}{2} + 0.866(2\pi x_1)\right]$$

Incorporating the linear part of $\underline{f}(\underline{x})$ into the system matrix, (4.57) is written as

$$\dot{\underline{x}} = \overline{A}\,\underline{x} + \underline{\phi}(\underline{x}) + B\,u \tag{4.58}$$

where

$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{f^*T_{1p}}{2H_1}(0.866)2\pi & -\frac{f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 \\ 0 & 0 & -\frac{1}{T_{t1}} & \frac{1}{T_{t1}} \\ 0 & -\frac{1}{R_1T_{gv1}} & 0 & -\frac{1}{T_{gv1}} \end{bmatrix} \tag{4.59}$$

and

$$\underline{\phi}(\underline{x}) = \begin{bmatrix} 0 \\ \frac{f^*T_{1p}}{2H_1}\,0.5\frac{(2\pi x_1)^2}{2} \\ 0 \\ 0 \end{bmatrix} \tag{4.60}$$

For the application of optimal control theory to the system given in (4.57) the performance index is taken as

$$J = \int_0^\infty \begin{bmatrix} \underline{x}^T & u^T \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \underline{x} \\ u \end{bmatrix} dt \tag{4.61}$$

where

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.62}$$

and R is a unit scalar.

Lukes' theory presented in Section 4.2 is now applied to this system. The following features are noted for the system under study:

(i) The matrix N of (4.9) reduces here to a 4-element null vector.

(ii) $\underline{g}(\underline{x},\underline{u})$ of (4.9) becomes zero here.

(iii) Corresponding to $\underline{f}(\underline{x},\underline{u})$ of (4.3) here we have

$$\underline{f}(\underline{x}) = \begin{bmatrix} 0 \\ \frac{f^* T_{1p}}{2H_1} 0.5(2\pi)^2 \left(\frac{x_1^2}{2} \right) \\ 0 \\ 0 \end{bmatrix} \qquad (4.63)$$

(iv) The matrix quadratic equation described in (4.13) reduces in this case to

$$Q + P_* \bar{A} + \bar{A}^T P_* - P_* B R^{-1} B^T P_* = 0 \qquad (4.64)$$

The truncated system is first taken up for study. It is checked up for controllability; then matrix Riccati equation (4.64) is solved for P_*. The same is computed as

$$\begin{bmatrix} 0.45777\times10^{0} & -0.62143\times10^{-1} & -0.18810\times10^{0} & -0.51001\times10^{-1} \\ -0.62143\times10^{-1} & 0.33741\times10^{0} & 0.26169\times10^{0} & 0.47288\times10^{-1} \\ -0.18810\times10^{0} & 0.26169\times10^{0} & 0.33747\times10^{0} & 0.75492\times10^{-1} \\ -0.51001\times10^{-1} & 0.47288\times10^{-1} & 0.75492\times10^{-1} & 0.18087\times10^{-1} \end{bmatrix}$$

Equation (4.18) is written here as

$$u_*^{(1)}(x) = K_* x \qquad (4.65)$$

and

$$K_* = -R^{-1}B^TP_* \tag{4.66}$$

where K_*, a 4-element row vector, is computed as

$$\begin{bmatrix} 0.63751 & -0.59110 & -0.94365 & -0.22608 \end{bmatrix}$$

The system matrix $A_* = (\bar{A} + B\,K_*)$ described by (4.21) is computed as

$$\begin{bmatrix} 0.0 & 0.10000\text{x}10^{1} & 0.0 & 0.0 \\ -0.32649\text{x}10^{1} & -0.49980\text{x}10^{-1} & 0.60000\text{x}10^{1} & 0.0 \\ 0.0 & 0.0 & -0.33333\text{x}10^{1} & 0.33333\text{x}10^{1} \\ 0.79688\text{x}10^{1} & -0.12597\text{x}10^{2} & -0.11796\text{x}10^{2} & -0.15326\text{x}10^{2} \end{bmatrix}$$

At this stage a check is made to see whether the system obeys the fundamental hypothesis, viz. stabilizability. For this purpose the eigen values of the above matrix $A_* = (\bar{A} + B\,K_*)$ are determined as:

1) $-\ 0.49353\text{x}10^{0} + j0.0$
2) $-\ 0.13345\text{x}10^{2} + j0.0$
3) $-\ 0.24352\text{x}10^{1} + j0.38322\text{x}10^{1}$
4) $-\ 0.24352\text{x}10^{1} - j0.38322\text{x}10^{1}$

The real parts for all the eigenvalues are in the left half plane and hence the system is stabilizable. The next step is to compute the nonlinear part of the feedback controller. Equation (4.30) for m=3, here simplifies to

$$A_*\underline{x}.J_x^{(3)}(\underline{x}) = -\underline{f}^{(2)}(\underline{x}).J_x^{(2)}(\underline{x}) \tag{4.67}$$

In (4.67) all the terms are known except $J^{(3)}(\underline{x})$ which is assumed to have the form

$$J^{(3)}(\underline{x}) = c_1x_1^3 + c_2x_1^2x_2 + c_3x_1x_2^2 + c_4x_1^2x_3 + c_5x_1x_3^2 + c_6x_1^2x_4$$
$$+ c_7x_1x_4^2 + c_8x_2^3 + c_9x_2^2x_3 + c_{10}x_2x_3^2 + c_{11}x_2^2x_4$$
$$+ c_{12}x_2x_4^2 + c_{13}x_3^3 + c_{14}x_3^2x_4 + c_{15}x_3x_4^2 + c_{16}x_4^3$$
$$+ c_{17}x_1x_2x_3 + c_{18}x_1x_2x_4 + c_{19}x_1x_3x_4 + c_{20}x_2x_3x_4 \quad .. \quad (4.68)$$

By performing the products as given in (4.67) and equating coefficients of like powers of $\underline{x}$ on either side, the computation reduces to that of the solution of a system of 20th order linear algebraic equations given by

$$\nu\cdot\underline{x} = \underline{n} \qquad (4.69)$$

where ν is a 20x20 matrix and $\underline{n}$ is a 20x1 vector. The description of the elements of ν and $\underline{n}$ are given in Appendix E. The elements of the matrix ν are constituted of the elements of the matrix A_* which is computed above; hence their numerical values are not given. Finally the elements of the solution vector $\underline{n}$ which are the same as c_1 to c_{20}, the coefficients in the expression for $J^{(3)}(\underline{x})$, are computed as

$c_1 = -0.78146\times10^0$ $c_5 = 0.38962\times10^{-1}$

$c_2 = 0.19132\times10^0$ $c_6 = 0.17074\times10^0$

$c_3 = -0.27938\times10^{-1}$ $c_7 = 0.26203\times10^{-2}$

$c_4 = 0.64460\times10^0$ $c_8 = -0.14959\times10^{-1}$

$c_9 = -0.25304\text{x}10^{-1}$

$c_{10} = -0.21228\text{x}10^{-1}$

$c_{11} = -0.20398\text{x}10^{-2}$

$c_{12} = -0.47398\text{x}10^{-3}$

$c_{13} = -0.77631\text{x}10^{-2}$

$c_{14} = -0.42168\text{x}10^{-2}$

$c_{15} = -0.84692\text{x}10^{-3}$

$c_{16} = -0.61400\text{x}10^{-4}$

$c_{17} = 0.84663\text{x}10^{-1}$

$c_{18} = 0.40938\text{x}10^{-1}$

$c_{19} = 0.24071\text{x}10^{-1}$

$c_{20} = -0.58479\text{x}10^{-2}$

$\underline{u}_*^{(2)}(\underline{x})$ is then computed from (4.31) with $l = 2$. The same in this case reduces to

$$\underline{u}_*^{(2)}(\underline{x}) = -\frac{1}{2} R^{-1}B^T J_x^{(3)}(\underline{x}) \tag{4.70}$$

$J_x^{(3)}(\underline{x})$ here is a 4-element column vector consisting of partial differentials of $J^{(3)}(\underline{x})$ with respect to x_1, x_2, x_3 and x_4 respectively, as its elements. On performing this partial differentiation and the matrix products in (4.70), we get

$$u_*^{(2)}(\underline{x}) = d_1x_1^2 + d_2x_1x_4 + d_3x_2^2 + d_4x_2x_4 + d_5x_3^2 + d_6x_3x_4 + d_7x_4^2 + d_8x_1x_2 + d_9x_1x_3 + d_{10}x_2x_3 \tag{4.71}$$

where the coefficients d_1 to d_{10} are calculated as

$d_1 = -0.10672\text{x}10^{1}$

$d_2 = -0.32753\text{x}10^{-1}$

$d_3 = 0.12749\text{x}10^{-1}$

$d_4 = 0.59248\text{x}10^{-2}$

$d_5 = 0.26355\text{x}10^{-1}$

$d_6 = 0.10587\text{x}10^{-1}$

$d_7 = 0.11513\text{x}10^{-2}$

$d_8 = -0.25586\text{x}10^{0}$

$d_9 = -0.15045\text{x}10^{0}$

$d_{10} = 0.36549\text{x}10^{-1}$

As stated before, the computations are stopped with $J^{(3)}(\underline{x})$ and $u_*^{(2)}(\underline{x})$. The optimum feedback control is now given by

$$\underline{u}_*(\underline{x}) = \underline{u}_*^{(1)}(\underline{x}) + \underline{u}_*^{(2)}(\underline{x}) \tag{4.72}$$

where $u_*^{(1)}(\underline{x})$ and $u_*^{(2)}(\underline{x})$ are calculated as given in (4.65) and (4.71) respectively. This expression for $\underline{u}_*(\underline{x})$ is used in the system equation (4.58) and the response of the state variable x_2 determined. Table 4.2 gives ten arbitrary sets of initial conditions that are taken for this purpose.

Table 4.2

4th order system - list of initial conditions.

Sl.No.	Initial conditions for			
	x_1	x_2	x_3	x_4
1	$0.01/2\pi$	0.0	0.0	0.0
2	$0.349/2\pi$	0.0	0.0	0.0
3	$0.715/2\pi$	0.0	0.0	0.0
4	$0.05/2\pi$	0.025	-0.02	-0.01
5	$0.10/2\pi$	-0.005	0.008	0.004
6	$0.15/2\pi$	0.03	-0.015	-0.001
7	$0.25/2\pi$	0.04	-0.018	0.008
8	$0.01/2\pi$	0.001	-0.02	0.0
9	$0.20/2\pi$	-0.008	0.01	-0.008
10	$0.30/2\pi$	-0.023	0.026	0.01

The responses obtained with the above initial conditions are given as continuous curves in Figures 4.2 to 4.11. In Subsection B below suboptimal responses will be obtained making use of a reduced model, for the same initial conditions and will be compared with the optimal responses obtained as above.

B. Suboptimal Regulation by Reduction

In Subsection A above, the optimal feedback control of the 4th order nonlinear system was determined by directly applying Lukes' method to the same. In this subsection the 4th order system is reduced to a 3rd order nonlinear model using the theory given in Section 4.4; Lukes' method is then applied to this 3rd order system to determine the optimal feedback controller for the same. This is used as a suboptimal controller for the original system.

The dynamics of the 4th order system is rewritten from (4.57) as

$$\dot{\underline{x}} = A\,\underline{x} + B\,u + \underline{f}(\underline{x}) \tag{4.73}$$

where $\underline{x}$ is a 4-vector, u is the scalar control and $\underline{f}(\underline{x})$ is a 4-element vector function. Now reduction of the above system to 3rd order is affected using the transformation $\underline{z} = C\,\underline{x}$ given in (4.34).

For the purpose of computing C, the maximum excursion of the angle $2\pi x_1$ is taken as, say 0.715 radian.

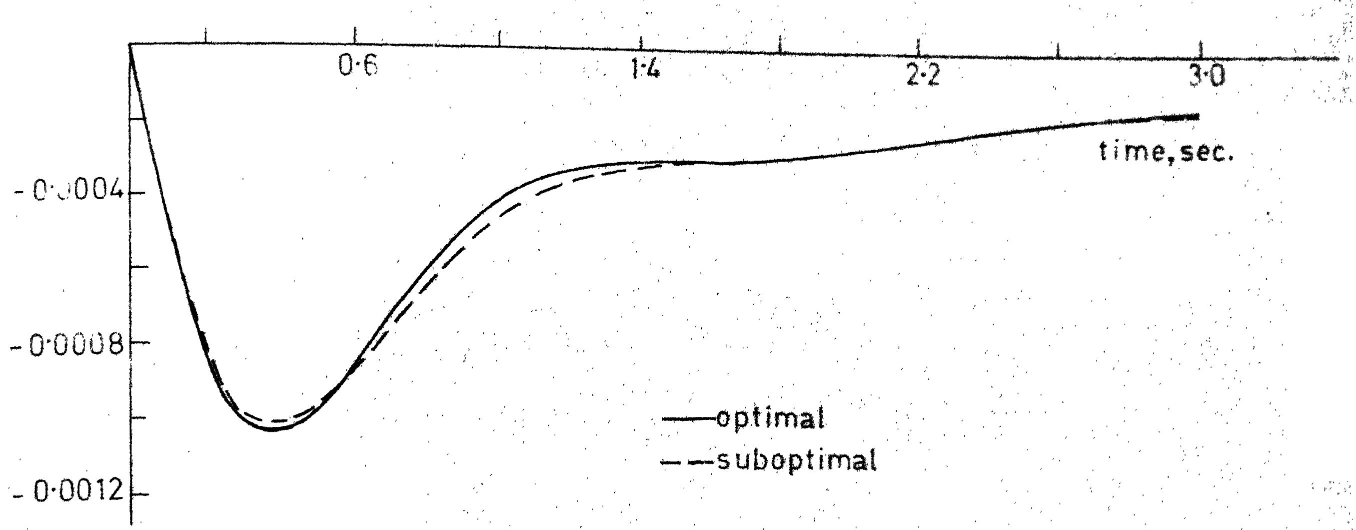


Fig. 4·2 Response of Δf with initial condition $x_1 = 0{\cdot}01/2\pi$, $x_2 = 0{\cdot}0$, $x_3 = 0{\cdot}0$, $x_4 = 0{\cdot}0$

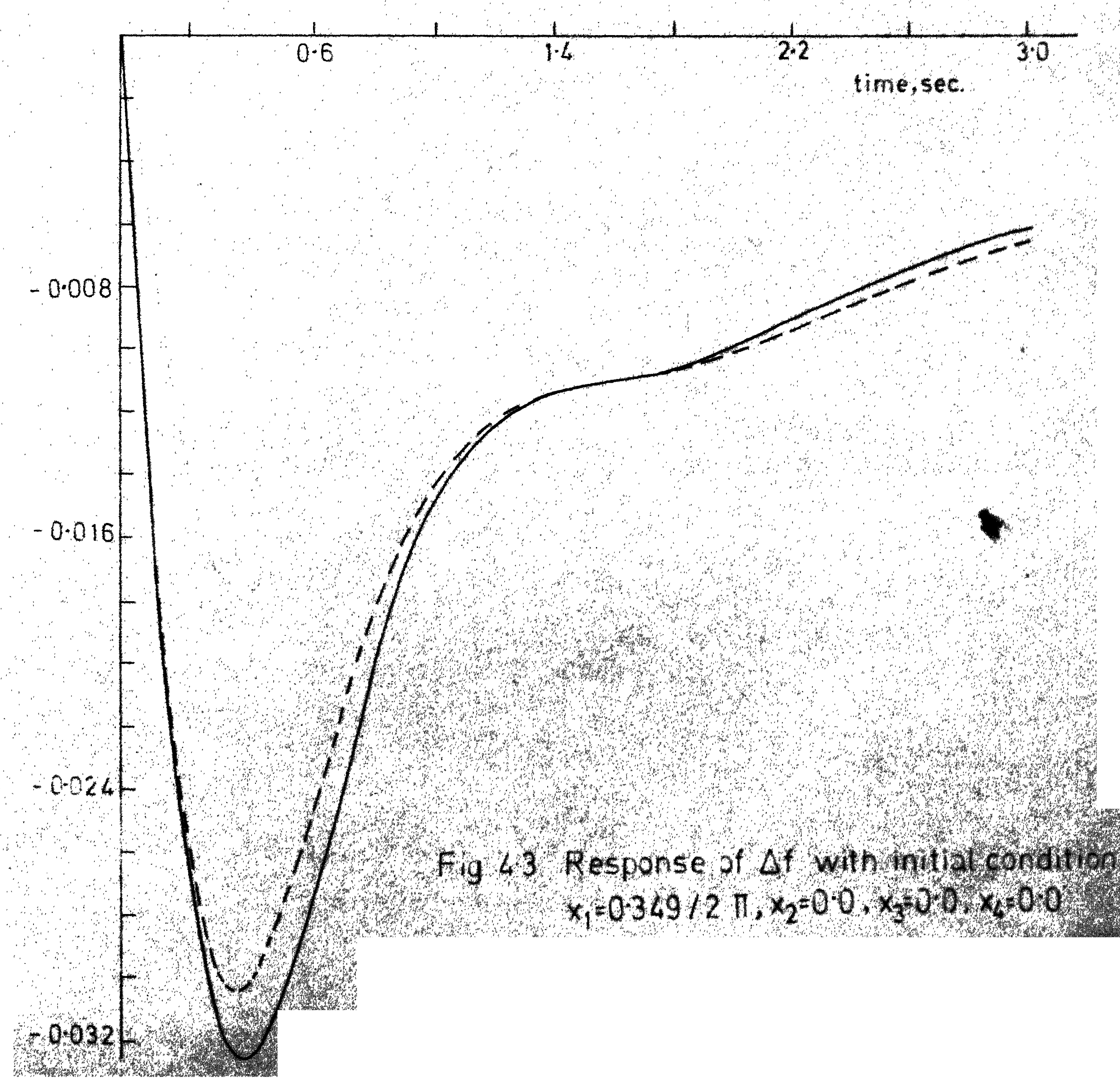


Fig 4·3 Response of Δf with initial condition $x_1 = 0{\cdot}349/2\pi$, $x_2 = 0{\cdot}0$, $x_3 = 0{\cdot}0$, $x_4 = 0{\cdot}0$

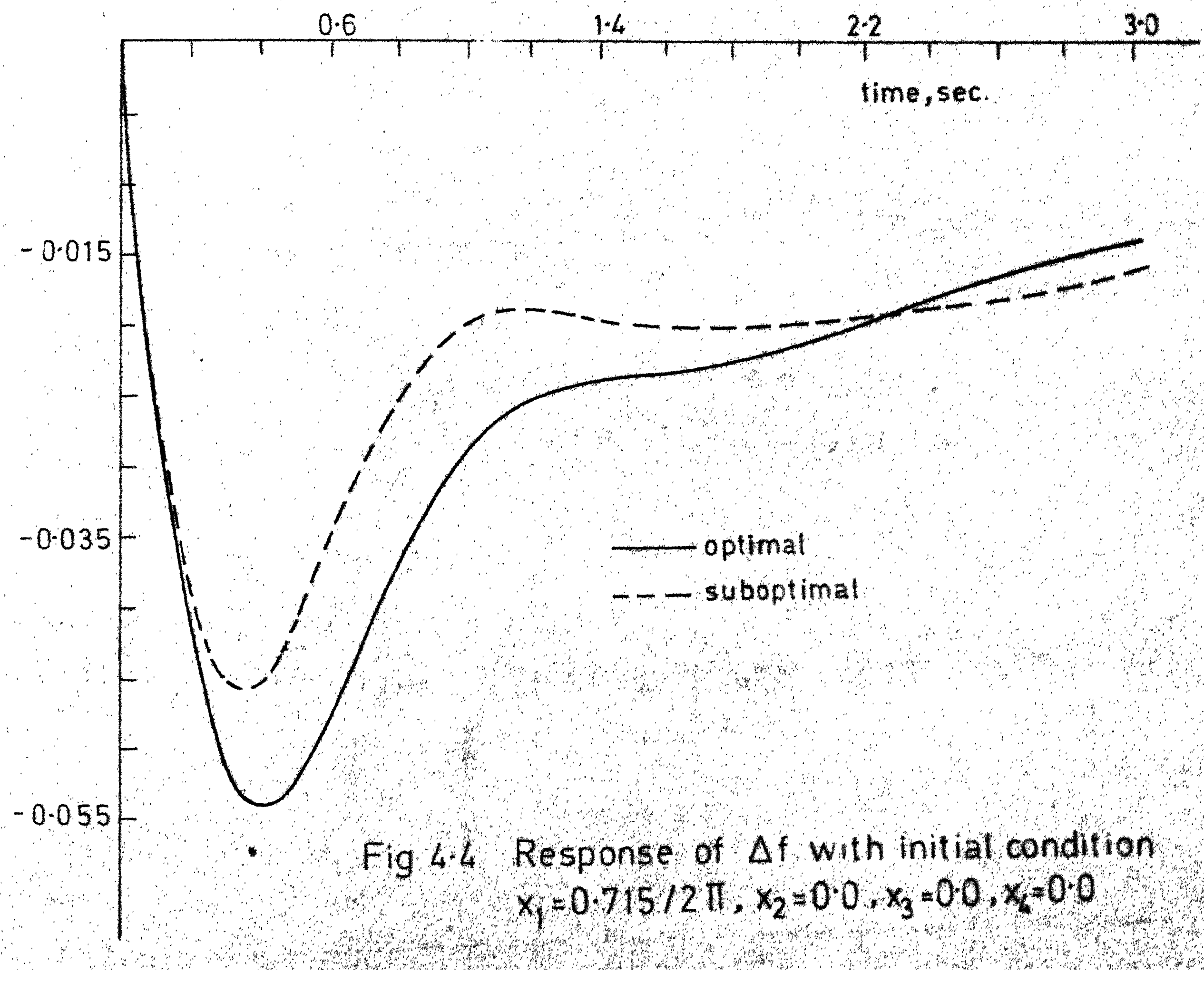


Fig 4·4 Response of Δf with initial condition $x_1 = 0{\cdot}715/2\pi$, $x_2 = 0{\cdot}0$, $x_3 = 0{\cdot}0$, $x_4 = 0{\cdot}0$

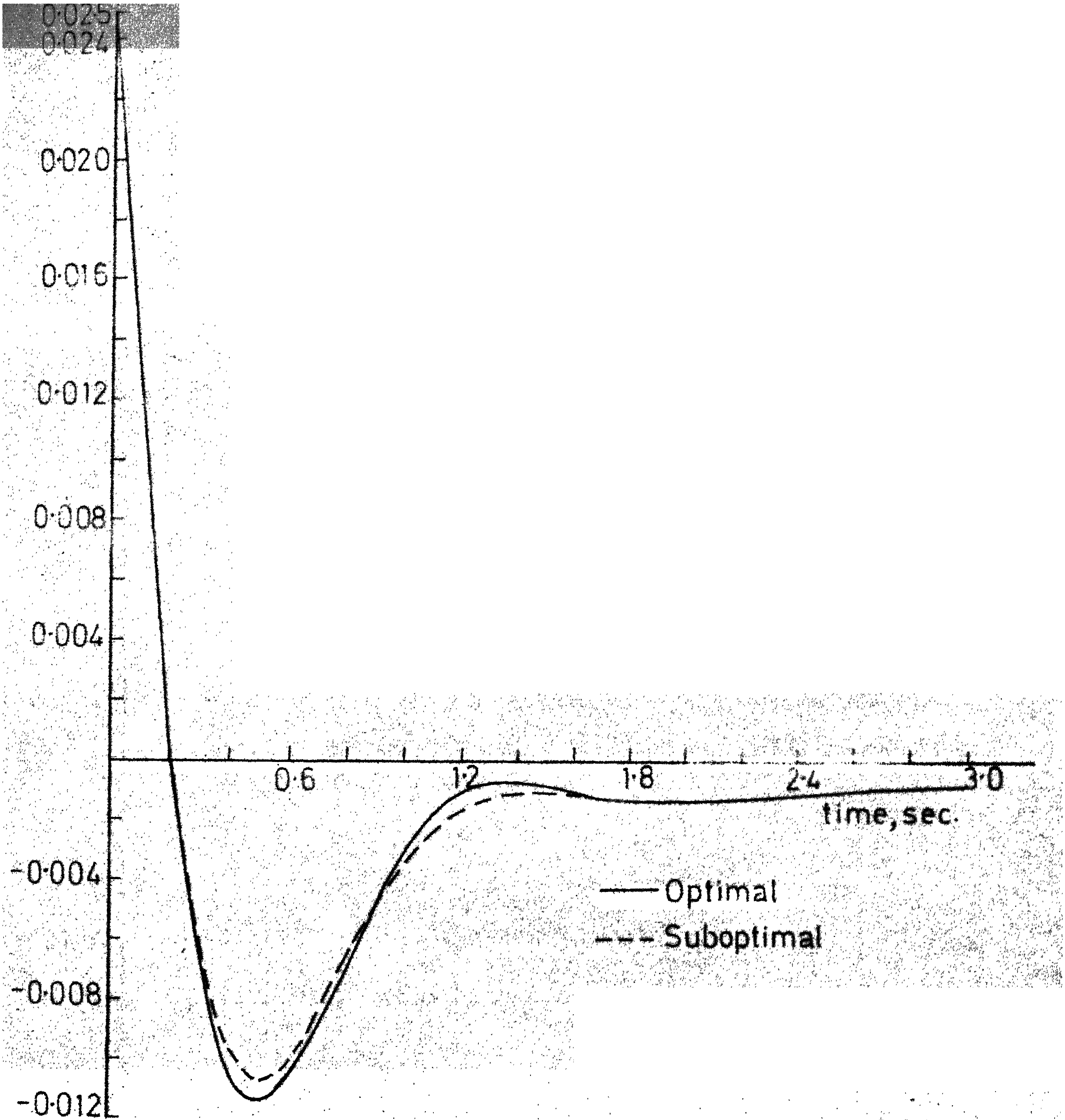


Fig. 4·5 Response of Δf with initial condition $x_1 = 0{\cdot}05/2\pi$, $x_2 = 0{\cdot}025$, $x_3 = -0{\cdot}02$, $x_4 = -0{\cdot}01$

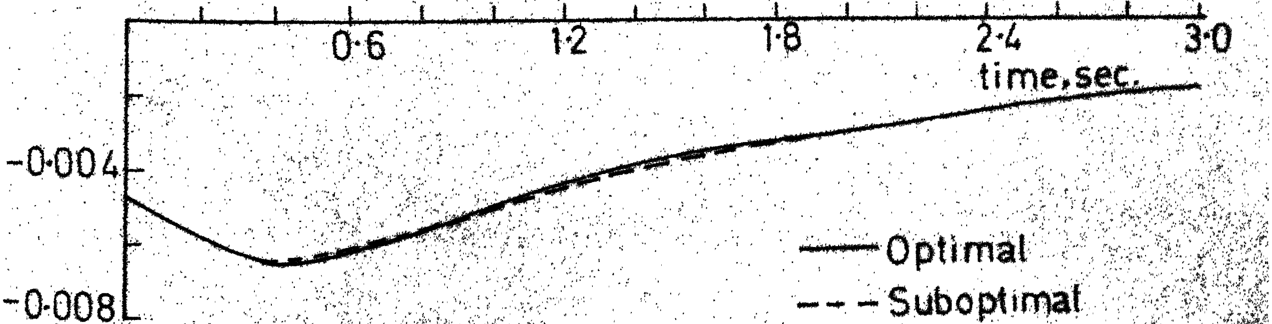


Fig. 4·6 Response of Δf with initial condition $x_1 = 0{\cdot}1/2\pi$, $x_2 = -0{\cdot}005$, $x_3 = 0{\cdot}008$, $x_4 = 0{\cdot}004$

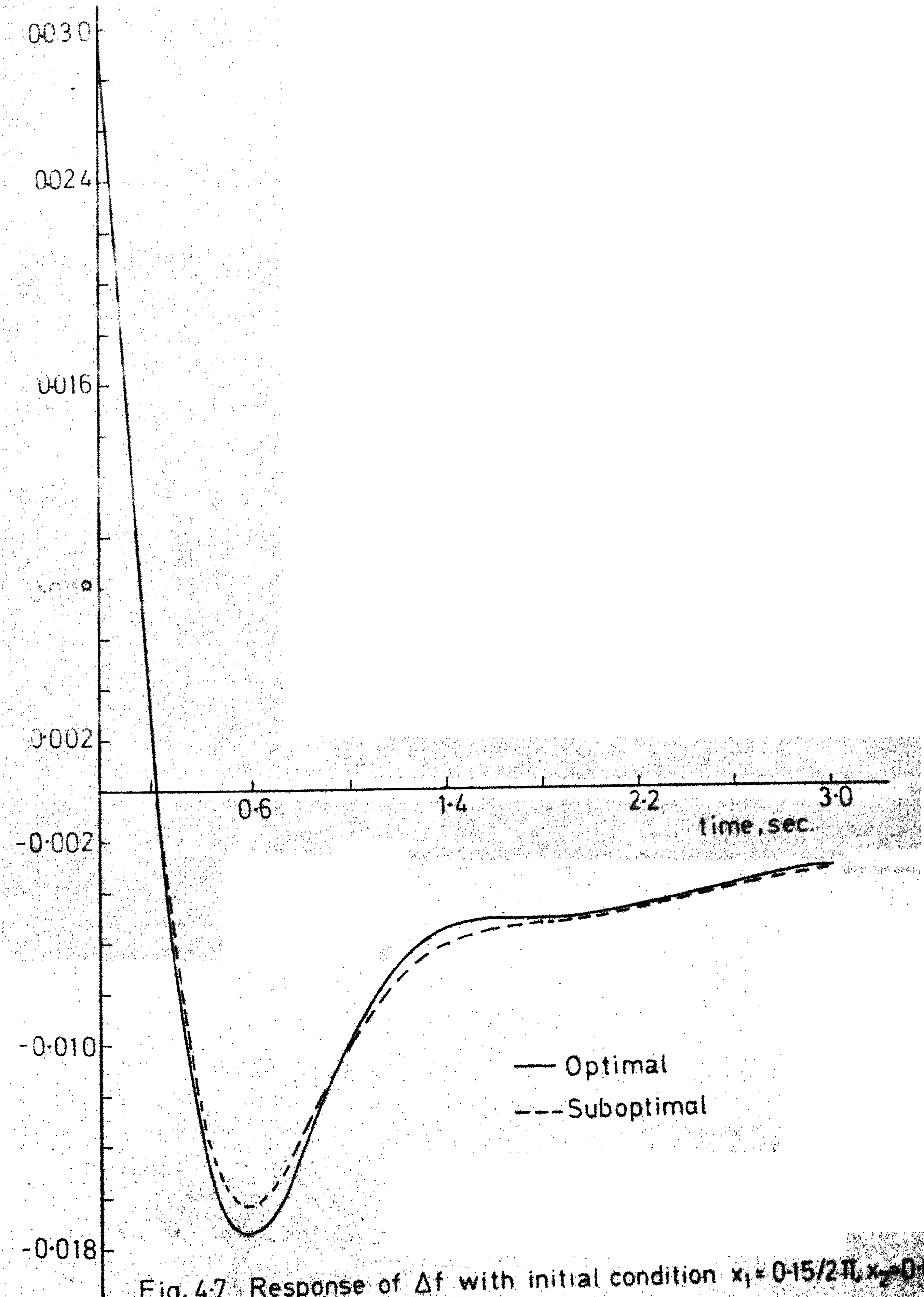


Fig. 4·7 Response of Δf with initial condition $x_1 = 0{\cdot}15/2\pi$, $x_2 = 0{\cdot}03$, $x_3 = -0{\cdot}015$, $x_4 = -0{\cdot}001$

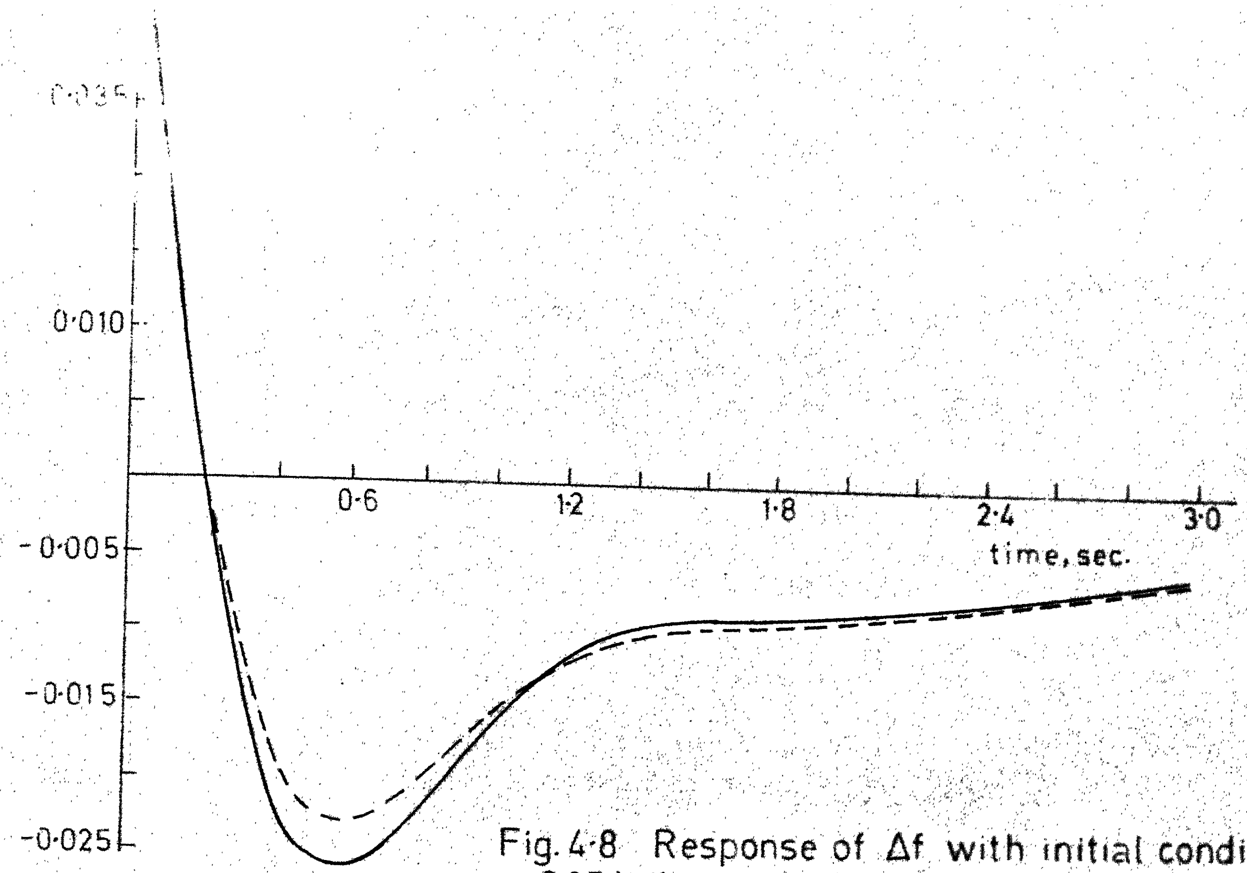


Fig. 4·8 Response of Δf with initial condition $x_1 = 0{\cdot}25/2\pi$, $x_2 = 0{\cdot}04$, $x_3 = -0{\cdot}018$, $x_4 = 0{\cdot}008$

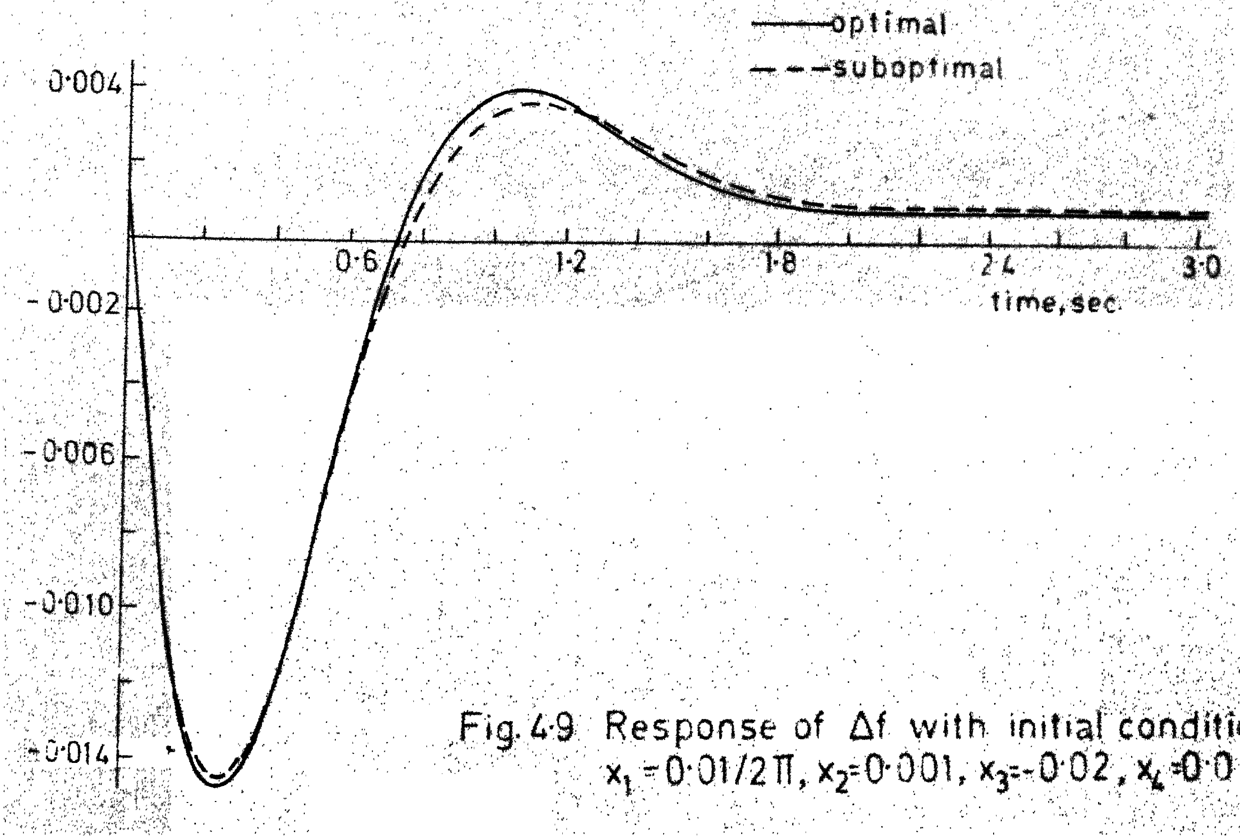


Fig. 4·9 Response of Δf with initial condition $x_1 = 0{\cdot}01/2\pi$, $x_2 = 0{\cdot}001$, $x_3 = -0{\cdot}02$, $x_4 = 0{\cdot}0$

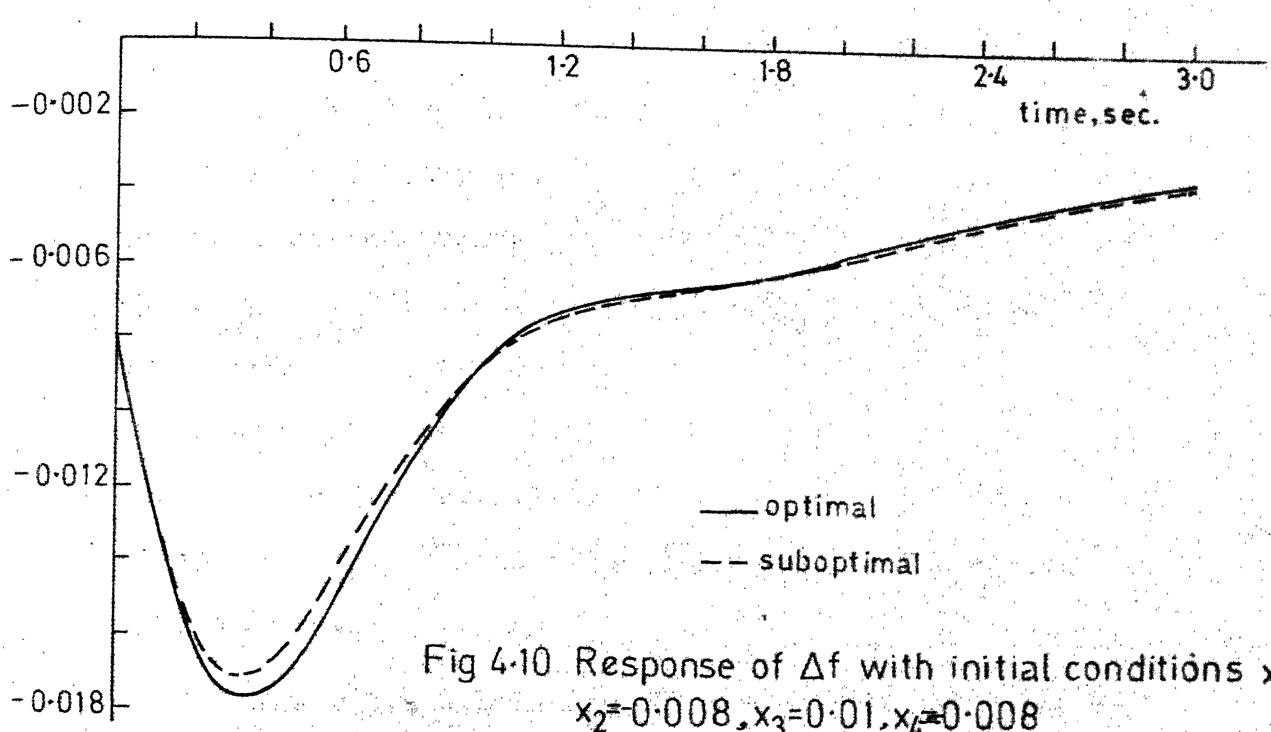


Fig 4·10 Response of Δf with initial conditions $x_1=0{\cdot}2/2\Pi$ $x_2=-0{\cdot}008$, $x_3=0{\cdot}01$, $x_4=0{\cdot}008$

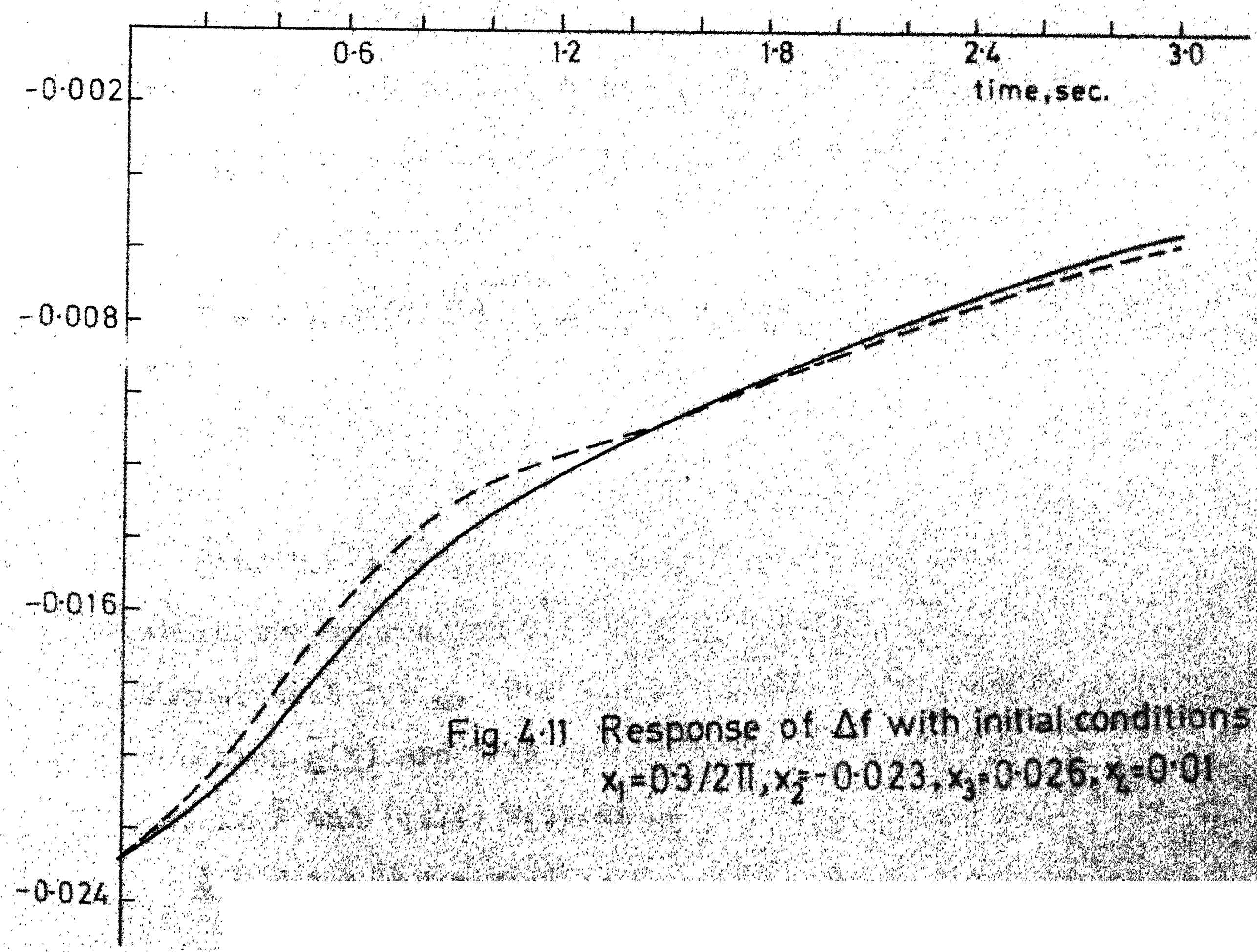


Fig. 4·11 Response of Δf with initial conditions $x_1=0{\cdot}3/2\Pi$, $x_2=-0{\cdot}023$, $x_3=0{\cdot}026$, $x_4=0{\cdot}01$

Figure 4.1 is made use of and the value of the nonlinear function $\boxed{0.5(\text{Cos}2\pi x_1 - 1.0) + 0.866\text{Sin}2\pi x_1}$ read off from the same corresponding to $2\pi x_1$ an angle which is midway between the range from 0 to 0.715 radian. This value is multiplied by $-\frac{f^*T_{1p} \times 2\pi}{2H_1}$ to get the linear term in x_1 which is then incorporated in the system matrix A to form matrix H. The rows of the transformation matric C now constitute of the eigenvectors of H^T corresponding to its three dominant eigenvalues.

After transformation the dynamics of the 3rd order system becomes

$$\dot{\underline{z}} = \bar{F}\,\underline{z} + g\,u + \bar{\underline{g}}(\underline{z}) \tag{4.74}$$

where $\underline{z}$ is a 3-vector, u is the scalar control as before and $\underline{g}(\underline{z})$ is a 3-element vector function of $\underline{z}$.

$\bar{F}$ is computed as

$$\bar{F} = C\,A\,C^T(C\,C^T)^{-1} \tag{4.75}$$

and

$$G = C\,B \tag{4.76}$$

Also

$$\bar{\underline{g}}(\underline{z}) = C\,\underline{f}(\underline{x}) \tag{4.77}$$

where the expression $C^T(C\,C^T)^{-1}\,\underline{z}$, given in (4.45) is substituted for $\underline{x}$. The linear terms in the vector function $\bar{\underline{g}}(\underline{z})$ are then incorporated in the system matrix $\bar{F}$ and (4.74) written as

$$\dot{\underline{z}} = F\,\underline{z} + G\,u + \underline{g}(\underline{z}) \tag{4.78}$$

where $\underline{g}(\underline{z})$ is now comprising of only second powers of the elements of $\underline{z}$. Here the matrices C, F and G are computed as

$$C = \begin{bmatrix} -0.33002 & 0.21296 & 0.71120 & 0.20279 \\ -0.30149 & -0.42007 & -0.17733 & 0.0 \\ 0.71632 & 0.24560 & 0.62736 & 0.18160 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.73514\text{x}10^{0} & 0.29014\text{x}10^{1} & -0.88118\text{x}10^{-1} \\ -0.30623\text{x}10^{1} & -0.78371\text{x}10^{0} & 0.17381\text{x}10^{0} \\ 0.86271\text{x}10^{-1} & -0.15337\text{x}10^{-1} & -0.10861\text{x}10^{1} \end{bmatrix}$$

$$G = \begin{bmatrix} 0.25349\text{x}10^{1} \\ 0.0 \\ 0.22700\text{x}10^{1} \end{bmatrix}$$

The performance index for the reduced system is given by

$$J = \int_{0}^{\infty} (\underline{z}^{T} Q_M \underline{z} + u^{T} R u)\, dt \qquad (4.79)$$

where Q_M is as given in (4.52) with Q as in (4.62) and is computed as

$$\begin{bmatrix} 0.81562\text{x}10^{-1} & -0.79954\text{x}10^{0} & -0.29835\text{x}10^{0} \\ -0.79954\text{x}10^{0} & 0.78379\text{x}10^{1} & 0.29247\text{x}10^{1} \\ -0.29835\text{x}10^{0} & 0.29247\text{x}10^{1} & 0.10913\text{x}10^{1} \end{bmatrix}$$

The system described by (4.78) and (4.79) is now solved by Lukes' method. As in the 4th order case, the truncated or linear part of the system is taken up first.

At this stage the controllability of the (F,G) pair is checked up. $u_*^{(1)}(\underline{z})$ is determined as

$$u_*^{(1)}(\underline{z}) = -R^{-1}\, G^T\, \hat{P}\, \underline{z} = \overline{K}\, \underline{z} \tag{4.80}$$

$\hat{P}$ is the solution of the matrix quadratic equation

$$Q_M + \hat{P}\, F + F^T\, \hat{P} - \hat{P}\, G\, R^{-1}\, G^T\, \hat{P} = 0 \tag{4.81}$$

Here $\hat{P}$ is computed as

$$\begin{bmatrix} 0.12585\text{x}10^{1} & -0.76779\text{x}10^{0} & -0.66578\text{x}10^{0} \\ -0.76779\text{x}10^{0} & 0.17583\text{x}10^{1} & 0.51315\text{x}10^{0} \\ -0.66578\text{x}10^{0} & 0.51315\text{x}10^{0} & 0.51866\text{x}10^{0} \end{bmatrix}$$

$\overline{K}$ is a 3 element row vector and is computed as

$$\begin{bmatrix} 0.16789\text{x}10^{1} & -0.78146\text{x}10^{0} & -0.51035\text{x}10^{0} \end{bmatrix}$$

As before $J_z^{(3)}(\underline{z})$ is determined by using the equation

$$F_*\, \underline{z}.J_z^{(3)}(\underline{z}) = -\underline{g}^{(2)}(\underline{z}).J_z^{(2)}(z) \tag{4.82}$$

where

$$F_* = (F + G\, \overline{K}) \tag{4.83}$$

Here F_* is computed as

$$\begin{bmatrix} -0.49910\text{x}10^{1} & 0.48824\text{x}10^{1} & 0.12056\text{x}10^{1} \\ -0.30623\text{x}10^{1} & -0.78371\text{x}10^{0} & 0.17381\text{x}10^{0} \\ -0.37248\text{x}10^{1} & 0.17586\text{x}10^{1} & 0.72432\text{x}10^{-1} \end{bmatrix}$$

The stability of the reduced system is determined by computing the eigenvalues of F_* given in (4.83) as

1) $-0.51514\text{x}10^{0} + j0.0$

2) $-0.25936\times10^{1} + j0.36374\times10^{1}$

3) $-0.25936\times10^{1} - j0.36374\times10^{1}$

All the above eigenvalues are having negative real parts; and hence the system is stabilizable.

Now, in (4.82) all the terms are known except $J_{z}^{(3)}(\underline{z})$. The following expression is assumed for $J^{(3)}(\underline{z})$.

$$J^{(3)}(z) = d_1 z_1^3 + d_2 z_2^3 + d_3 z_3^3 + d_4 z_1^2 z_2 + d_5 z_1 z_2^2 + d_6 z_1^2 z_3 + d_7 z_1 z_3^2 + d_8 z_2^2 z_3 + d_9 z_2 z_3^2 + d_{10} z_1 z_2 z_3 \quad (4.84)$$

$J_{z}^{(3)}(\underline{z})$ is the column vector function consisting of the partial differentials of $J^{(3)}(\underline{z})$ with respect to z_1, z_2 and z_3, as its elements. The vector $\underline{g}^{(2)}(\underline{z})$ is that part of the vector $\bar{\underline{g}}(\underline{z})$ which consists of only second order terms. By performing the products given in (4.82) and equating like coefficients on either side, the solution reduces to that of solving a set of linear algebraic equations of 10th order viz.,

$$\Lambda \underline{m} = \underline{\varepsilon} \quad (4.85)$$

where Λ is a (10x10) matrix and $\underline{\varepsilon}$ is a (10x1) vector. $\underline{m}$ is a (10x1) column vector to be determined. Its elements are only the coefficients d_1 to d_{10} in the

expression for $J^{(3)}(z)$. Here Λ is described as

$$\Lambda = \begin{bmatrix} 3f_{11} & 0 & 0 & f_{21} & 0 & f_{31} & 0 & 0 & 0 & 0 \\ 0 & 3f_{22} & 0 & 0 & f_{12} & 0 & 0 & f_{32} & 0 & 0 \\ 0 & 0 & 3f_{33} & 0 & 0 & 0 & f_{13} & 0 & f_{23} & 0 \\ 3f_{12} & 0 & 0 & (2f_{11}+f_{22}) & 2f_{21} & f_{32} & 0 & 0 & 0 & f_{31} \\ 0 & 2f_{21} & 0 & 2f_{12} & (f_{11}+2f_{22}) & 0 & 0 & f_{31} & 0 & f_{32} \\ 3f_{13} & 0 & 0 & f_{23} & 0 & (2f_{11}+f_{33}) & 2f_{31} & 0 & 0 & f_{21} \\ 0 & 0 & 3f_{31} & 0 & 0 & 2f_{13} & (f_{11}+2f_{33}) & 0 & f_{21} & f_{23} \\ 0 & 3f_{23} & 0 & 0 & f_{13} & 0 & 0 & (2f_{22}+f_{33}) & 2f_{32} & f_{12} \\ 0 & 0 & 3f_{32} & 0 & 0 & 0 & f_{12} & 2f_{23} & (f_{22}+2f_{33}) & f_{13} \\ 0 & 0 & 0 & 2f_{13} & 2f_{23} & 2f_{12} & 2f_{32} & 2f_{21} & 2f_{31} & (f_{11}+f_{22}+f_{33}) \end{bmatrix} \quad \ldots (4.86)$$

where f_{ij}'s are the elements of the matrix F_* of (4.83).

The same is computed as

$$
\begin{bmatrix}
-0.14973\times10^{2} & 0.0 & 0.0 & -0.30623\times10^{1} & 0.0 & -0.37248\times10^{1} & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.23511\times10^{1} & 0.0 & 0.0 & 0.48824\times10^{1} & 0.0 & 0.0 & 0.17586\times10^{1} & 0.0 & 0.0 \\
0.0 & 0.0 & 0.21730\times10^{0} & 0.0 & 0.0 & 0.0 & 0.12056\times10^{1} & 0.0 & 0.17381\times10^{0} & 0.0 \\
0.14647\times10^{2} & 0.0 & 0.0 & -0.10766\times10^{2} & -0.61246\times10^{1} & 0.17586\times10^{1} & 0.0 & 0.0 & 0.0 & -0.37248\times10^{1} \\
0.0 & -0.91869\times10^{1} & 0.0 & 0.97647\times10^{1} & -0.65585\times10^{1} & 0.0 & 0.0 & -0.37248\times10^{1} & 0.0 & 0.17586\times10^{1} \\
0.36168\times10^{1} & 0.0 & 0.0 & 0.17381\times10^{0} & 0.0 & -0.99097\times10^{1} & -0.74496\times10^{1} & 0.0 & 0.0 & -0.30623\times10^{1} \\
0.0 & 0.0 & -0.11174\times10^{2} & 0.0 & 0.0 & 0.24112\times10^{1} & -0.48462\times10^{1} & 0.0 & -0.30623\times10^{1} & -0.17381\times10^{0} \\
0.0 & 0.52144\times10^{0} & 0.0 & 0.0 & 0.12056\times10^{1} & 0.0 & 0.0 & -0.14950\times10^{1} & 0.35171\times10^{1} & 0.48824\times10^{1} \\
0.0 & 0.0 & 0.52757\times10^{1} & 0.0 & 0.0 & 0.0 & 0.48824\times10^{1} & 0.34763\times10^{0} & -0.63885\times10^{0} & 0.12056\times10^{1} \\
0.0 & 0.0 & 0.0 & 0.24112\times10^{1} & 0.34763\times10^{0} & 0.97647\times10^{1} & 0.35171\times10^{1} & -0.61246\times10^{1} & -0.74496\times10^{1} & -0.57023\times10^{1}
\end{bmatrix}
$$

For describing the vector $\underline{\varepsilon}$ the following equalities are assumed. Let

$$f_{ta} = p_{11}c_{12} + p_{12}c_{22} + p_{13}c_{32}$$
$$f_{tb} = p_{12}c_{12} + p_{22}c_{22} + p_{23}c_{32} \qquad (4.87)$$
$$f_{tc} = p_{13}c_{12} + p_{23}c_{22} + p_{33}c_{32}$$

where c_{ij}'s are the elements of the transformation matrix C and let

$$b_x = -(f^* \, T_{1p} \times 0.5 \times 4\pi^2)/(4 \times H_1) \qquad (4.88)$$

Also let the matrix $\hat{H} = C^T(C\,C^T)^{-1}$ and let $\hat{h}_{ij}$'s be its elements, then

$$\varepsilon_1 = 2b_x(\hat{h}_{11})^2 f_{ta}$$
$$\varepsilon_2 = 2b_x(\hat{h}_{12})^2 f_{tb}$$
$$\varepsilon_3 = 2b_x(\hat{h}_{13})^2 f_{tc}$$
$$\varepsilon_4 = 4b_x(\hat{h}_{11}\,\hat{h}_{12}\,f_{ta} + 2b_x(\hat{h}_{11})^2 f_{tb}$$
$$\varepsilon_5 = 2b_x(\hat{h}_{12})^2 f_{ta} + 4b_x\,\hat{h}_{11}\,\hat{h}_{12}\,f_{tb}$$
$$\varepsilon_6 = 4b_x\,\hat{h}_{11}\,\hat{h}_{13}\,f_{ta} + 2b_x(\hat{h}_{11})^2 f_{tc}$$
$$\varepsilon_7 = 2b_x(\hat{h}_{13})^2 f_{ta} + 4b_x\,\hat{h}_{11}\,\hat{h}_{13}\,f_{tc}$$
$$\varepsilon_8 = 4b_x\,\hat{h}_{12}\,\hat{h}_{13}\,f_{tb} + 2b_x(\hat{h}_{12})^2 f_{tc}$$
$$\varepsilon_9 = 2b_x(\hat{h}_{13})^2 f_{tb} + 4b_x\,\hat{h}_{12}\,\hat{h}_{13}\,f_{tc}$$
$$\varepsilon_{10} = 4b_x(\hat{h}_{12}\,\hat{h}_{13}\,f_{ta} + \hat{h}_{11}\,\hat{h}_{13}\,f_{tb} + \hat{h}_{11}\,\hat{h}_{12}\,f_{tc}) \quad .. \quad (4.89)$$

These are computed as

$$\varepsilon_1 = -0.40294\text{x}10^1 \qquad \varepsilon_6 = 0.11663\text{x}10^2$$
$$\varepsilon_2 = 0.23146\text{x}10^0 \qquad \varepsilon_7 = -0.10703\text{x}10^2$$
$$\varepsilon_3 = 0.30109\text{x}10^1 \qquad \varepsilon_8 = 0.31358\text{x}10^1$$
$$\varepsilon_4 = 0.87561\text{x}10^1 \qquad \varepsilon_9 = 0.11070\text{x}10^2$$
$$\varepsilon_5 = -0.27312\text{x}10^1 \qquad \varepsilon_{10} = -0.19712\text{x}10^2$$

The elements of the solution vector which are the same as the coefficients of the expression for $J^{(3)}(\underline{z})$ are computed as

$$d_1 = 0.11032\text{x}10^1 \qquad d_6 = -0.31170\text{x}10^1$$
$$d_2 = 0.14259\text{x}10^0 \qquad d_7 = 0.27200\text{x}10^1$$
$$d_3 = -0.69438\text{x}10^0 \qquad d_8 = -0.34833\text{x}10^0$$
$$d_4 = -0.28688\text{x}10^0 \qquad d_9 = -0.67608\text{x}10^0$$
$$d_5 = 0.24154\text{x}10^0 \qquad d_{10} = 0.94776\text{x}10^0$$

$u_*^{(2)}(\underline{z})$ is computed from (4.31) with $l = 2$. The same in this case reduces to

$$u_*^{(2)}(\underline{z}) = -\tfrac{1}{2}R^{-1}\ G^T\ J_z^{(3)}(\underline{z}) \qquad (4.90)$$

On performing the matrix products in (4.90) we get the second order control function as

$$u_*^{(2)}(\underline{z}) = e_1 z_1^2 + e_2 z_1 z_2 + e_3 z_2^2 + e_4 z_1 z_3 + e_5 z_3^2 + e_6 z_2 z_3 \qquad .. \ (4.91)$$

where

$$e_1 = -0.65698\text{x}10^0 \qquad e_4 = 0.17269\text{x}10^1$$
$$e_2 = -0.34848\text{x}10^0 \qquad e_5 = -0.10832\text{x}10^1$$
$$e_3 = 0.89218\text{x}10^{-1} \qquad e_6 = 0.33343\text{x}10^0$$

The optimal feedback control for the reduced system is given by

$$u_*(\underline{z}) = u_*^{(1)}(\underline{z}) + u_*^{(2)}(\underline{z}) \tag{4.92}$$

Now substituting $C\ \underline{x}$ for $\underline{z}$ in the expressions for $u_*^{(1)}(\underline{z})$ and $u_*^{(2)}(\underline{z})$ of (4.92) we get an expression in terms of $\underline{x}$, say $\underline{w}(\underline{x})$, which becomes a suboptimal controller for the original 4th order system. The same is given by

$$\underline{w}(\underline{x}) = K_1\ \underline{x} + K_2(\underline{x}) \tag{4.93}$$

Here $K_1 = \bar{K}\ C$ is computed as

$$\begin{bmatrix} 0.68405\text{x}10^0 & -0.56047\text{x}10^0 & -0.10124\text{x}10^1 & -0.24779\text{x}10^0 \end{bmatrix}$$

and is a 4-element row vector.

With the same 10 numbers arbitrary initial conditions given in Table 4.1 of Subsection A the response of the state variable x_2 is determined. The curves so obtained are given as dashed ones in Figures 4.2 to 4.11.

The second order performance index $J^{(2)}(\underline{x})$ is computed for two of the initial conditions using both optimal and suboptimal control, and is given in Table 4.3.

Table 4.3

4th order system - performance indices.

Initial Condition	$J^{(2)}(\underline{x})$ with optimal control	$J^{2}(\underline{x})$ with sub-optimal control
$x_1 = 0.01/2\pi$ $x_2 = 0.001$ $x_3 = -0.02$ $x_4 = 0.0$	0.13766×10^{-3}	0.13900×10^{-3}
$x_1 = 0.2/2\pi$ $x_2 = -0.008$ $x_3 = 0.01$ $x_4 = -0.008$	0.40186×10^{-3}	0.38308×10^{-3}

It is seen from Table 4.3 that for the first set of initial conditions, the suboptimal index $J^{(2)}(\underline{x})$ is higher than the optimal one by a very small percentage (0.96%). This shows that the degradation in performance with suboptimal control is negligible and that the reduced model correctly represents the original system. But for the second set of initial conditions, the suboptimal index $J^{(2)}(\underline{x})$ is slightly less than the optimal one. This can be explained as follows:

As can be seen from the corresponding figure (Figure 4.10) the two responses are quite close to each other. To correctly compare the performance index figures under this circumstance, it is proper to compute $J(\underline{x})$ only instead of $J^{(2)}(\underline{x})$ in both cases.

4.6 CONCLUSIONS

In this chapter a method for the suboptimal regulation of a multiarea LFC system is presented using model reduction and Lukes' method for regulation of nonlinear systems. The method is applied to a 4th order, single area LFC system to construct a suboptimal controller for the same. The suboptimal controller so obtained is compared with the optimal one which is determined by directly applying Lukes' method to the 4th order system.

It is seen from the responses drawn in Figures 4.2 to 4.11 for the ten arbitrary sets of initial conditions, that the suboptimal response is very close to the optimal one. This shows that the suboptimal controller is very satisfactory. The order of the matrix Riccati equation as well as that of the linear system of algebraic equations solved with the reduced model, serve to demonstrate the drastic reduction in computation accruing out of model reduction.

The above results go to show that considerable saving in computational effort can be derived without much sacrifice in accuracy, by applying the techniques presented in this chapter to the large signal model of a multiarea LFC system. Such a study is attempted in Chapter 5.

CHAPTER 5

COMPUTATION OF SUBOPTIMAL CONTROLLERS FOR A TWO-AREA SYSTEM

In this chapter suboptimal feedback controllers are designed for the large signal model of a two-area LFC system using the model reduction technique as well as Lukes' method[7] for optimal regulation presented in Chapter 4. Two kinds of reduction of the original 8th order system are attempted: reduction to 5th order and then to 3rd order. In both cases the improvement in response as well as that in performance over those obtained by applying linear suboptimal controllers are evaluated. Figure 5.1 gives the block diagram of a single area in the large signal model of a two-area LFC system.

5.2 TWO-AREA NONLINEAR LFC SYSTEM

The dynamics of the two-area LFC system with a step load disturbance in Area 1 is described by the following differential equations:

$$\frac{d}{dt}\left(\int \Delta f_1 \, dt\right) = \Delta f_1 \tag{5.1}$$

$$\frac{d}{dt}\Delta f_1 = -\frac{f^*D_1}{2H_1}\Delta f_1 + \frac{f^*}{2H_1}\Delta P_{g1} - \frac{f^*}{2H_1}\Delta P_{tie1} - \frac{f^*}{2H_1}\Delta P_{d1} \tag{5.2}$$

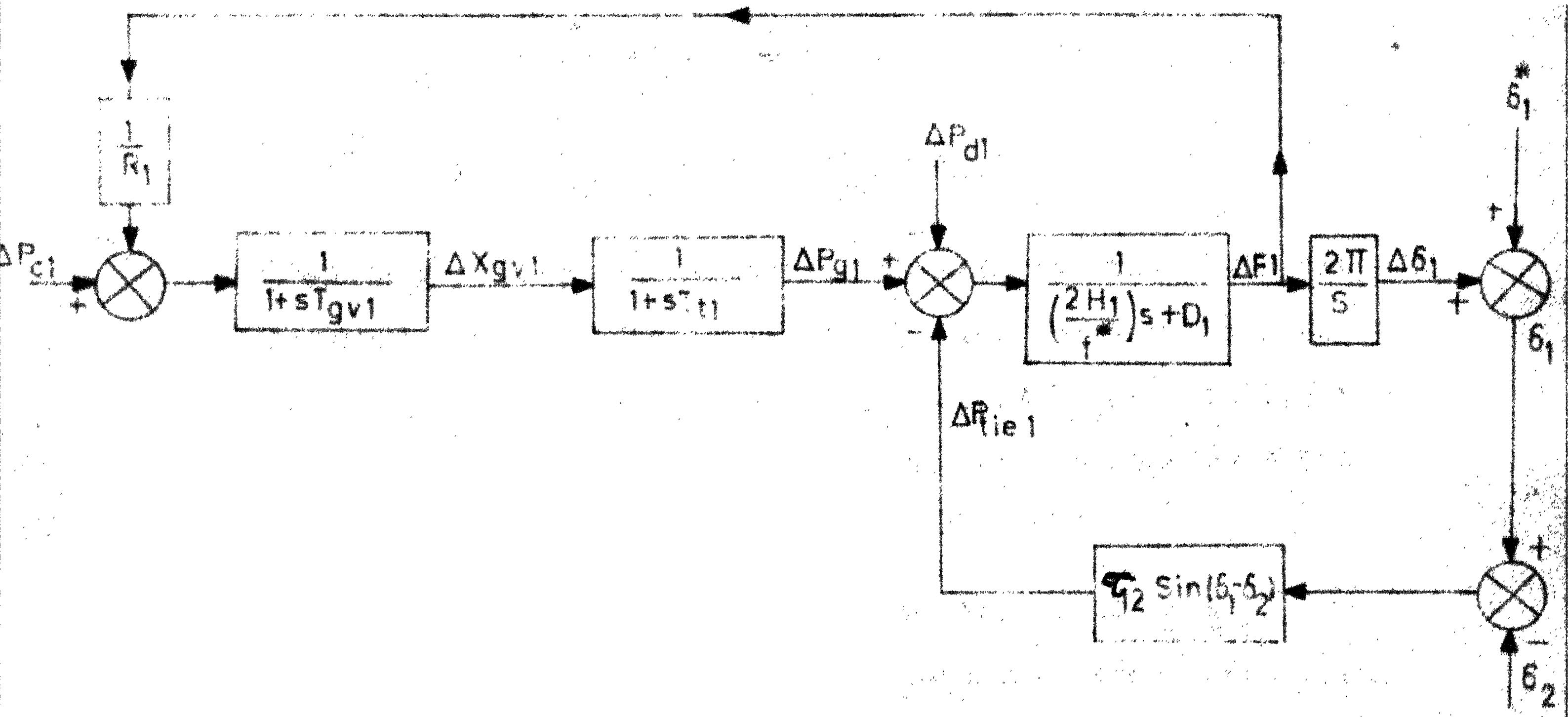


Fig 5·1 Block diagram of a single area in a two-area system (large-signal model)

$$\frac{d}{dt}\Delta P_{g1} = -\frac{1}{T_{t1}}\Delta P_{g1} + \frac{1}{T_{t1}}\Delta X_{gv1} \tag{5.3}$$

$$\frac{d}{dt}\Delta X_{gv1} = -\frac{1}{T_{gv1}R_1}\Delta f_1 - \frac{1}{T_{gv1}}\Delta X_{gv1} \tag{5.4}$$

$$\frac{d}{dt}\left(\int \Delta f_2 dt\right) = \Delta f_2 \tag{5.5}$$

$$\frac{d}{dt}\Delta f_2 = -\frac{f^*D_2}{2H_2}\Delta f_2 + \frac{f^*}{2H_2}\Delta P_{g2} - \frac{f^*}{2H_2}\Delta P_{tie2} \tag{5.6}$$

$$\frac{d}{dt}\Delta P_{g2} = -\frac{1}{T_{t2}}\Delta P_{g2} + \frac{1}{T_{t2}}\Delta X_{gv2} \tag{5.7}$$

$$\frac{d}{dt}\Delta X_{gv2} = -\frac{1}{T_{gv2}R_2}\Delta f_2 - \frac{1}{T_{gv2}}\Delta X_{gv2} \tag{5.8}$$

A state variable representation is made for the above set of simultaneous differential equations, the variables to be considered being $\int\Delta f_1 dt$, Δf_1, ΔP_{g1}, ΔX_{gv1}, $\int \Delta f_2 dt$, Δf_2, ΔP_{g2} and ΔX_{gv2}. Here the notations are the same as those used in Appendix A.

In (5.2) and (5.6) the terms ΔP_{tie1} and ΔP_{tie2} are to be expressed in terms of the state variables considered. This is accomplished as follows:

ΔP_{tie1} = change in tie line power of Area 1

$$= \tau_{12}\, \text{Sin}(\delta^*_{12} + \Delta\delta_{12}) - \tau_{12}\, \text{Sin}\,\delta^*_{12} \tag{5.9a}$$

where

$$\tau_{12} = \frac{|V_1|\,|V_2|}{X_{12}\, P_{r1}}$$ and δ^*_{12} is the nominal tieline angle before disturbance occurs. Equation (5.9à) can be written as

$$\Delta P_{tie1} = \tau_{12}\left[\text{Sin}\,\delta^*_{12}(\text{Cos}\,\Delta\delta_{12} - 1) + \text{Cos}\,\delta^*_{12}\,\text{Sin}\,\Delta\delta_{12}\right] \quad .. \tag{5.9b}$$

Using the relation

$$\Delta\delta_{12} = 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt) \qquad (5.10)$$

equation (5.9b) can be written as

$$\Delta P_{tie1} = \tau_{12}\{\sin\delta^*_{12}\left[\cos 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt) - 1\right]$$
$$+ \cos\delta^*_{12}\sin 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt)\} \qquad (5.11)$$

Using the relation $\Delta P_{tie2} = a_{12}\,\Delta P_{tie\,1}$ where $a_{12} = -P_{r1}/P_{r2}$ and P_{r1}, P_{r2} are the base power in Areas 1 and 2, -- we have

$$\Delta P_{tie2} = a_{12}\,\tau_{12}\{\sin\delta^*_{12}\left[\cos 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt) - 1\right]$$
$$+ \cos\delta^*_{12}\sin 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt)\} \qquad (5.12)$$

Substituting (5.11) and (5.12) for ΔP_{tie1} and ΔP_{tie2} in (5.2) and (5.6) respectively and also by redefining the state variables in terms of their steady state values to eliminate the term $-\frac{f^*\Delta P_{d1}}{2H_1}$ in (5.2) the dynamics of the two-area LFC system can be expressed in state variable form as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \underline{f}(\underline{x}) \qquad (5.13)$$

where A =

$$\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{T_{gv1}R_1} & 0 & \frac{-1}{T_{gv1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-f^*D_2}{2H_2} & \frac{f^*}{2H_2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{gv2}R_2} & 0 & \frac{-1}{T_{gv2}}
\end{bmatrix} \qquad (5.14)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{gv2}} \end{bmatrix}^T$$

Also

$$f_1(\underline{x}) = - \frac{f^*}{2H_1} \tau_{12} \{ 0.5 [\text{Cos}2\pi (x_1 - x_5) - 1] + 0.866 \text{Sin} 2\pi (x_1 - x_5) \}$$

$$f_2(x) = - \frac{f^*}{2H_2} a_{12} \tau_{12} \{ 0.5 [\text{Cos}2\pi (x_1 - x_5) - 1] + 0.866 \text{Sin} 2\pi (x_1 - x_5) \}$$

$$.. \quad (5.16)$$

The other elements of the vector function $\underline{f}(\underline{x})$ being zero. The expression in the brackets $\{ \}$ in both cases is the same and can be denoted as, say, $\phi(\underline{x})$. The function $\left[\frac{\phi(\underline{x})}{2\pi(x_1-x_5)}\right]$ is already plotted versus $2\pi(x_1-x_5)$ in Figure 4.1. This fact will be made use of in Section 5.3 below.

For posing the above system as a control problem, the performance to be minimized is taken as

$$J = \int_0^{\infty} (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u}) \, dt \qquad (5.17)$$

5.3 SUBOPTIMAL REGULATION USING MODEL REDUCTION

The theory for reduction of a two-area LFC system is given in Section 4.4. The next step viz., the regulation of the reduced (5th order) nonlinear system will be considered in this section. The computational results in respect of reduction to a 5th order model are dealt with in Section 5.4 and in respect of reduction to 3rd order, in Section 5.5.

The starting point for the regulation of the reduced (5th order) system are equations (4.50) and (4.51) which are reproduced below for convenience:

$$\dot{\underline{z}} = F\,\underline{z} + G\,\underline{u} + \underline{g}(\underline{z}) \tag{5.18}$$

$$J_M = \int_o^{\infty} (\underline{z}^T Q_M\,\underline{z} + \underline{u}^T R\,\underline{u})dt \tag{5.19}$$

The initial condition is given by

$$\underline{z}(o) = C\,\underline{x}(o) \tag{5.20}$$

Equation (5.19) can be written as

$$J_M = \int_o^{\infty} \begin{bmatrix} \underline{z}^T & \underline{u}^T \end{bmatrix} \begin{bmatrix} Q_M & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \underline{z} \\ \underline{u} \end{bmatrix} dt \tag{5.21}$$

Equations (5.18), (5.20) and (5.21) constitute the control problem for the 5th order model. It is assumed that the system described herein obeys the four basic assumptions made for the application of Lukes' method of regulation (Section 4.2). Now we shall apply the theory for optimal regulation to this control problem.

The first step is to solve the truncated or linear part of the system and corresponding quadratic performance index for obtaining a linear feedback control. Thus in (5.18) the linear part is written as

$$\dot{\underline{z}} = F\,\underline{z} + G\,\underline{u} \tag{5.22}$$

Now (5.22) together with (5.20) and (5.21) constitutes the control problem for the truncated system. The

controllability of the (F, G) pair is checked up. For determining the optimal feedback control parameters for this problem, the matrix Riccati equation described in (4.22) is solved which in this case reduces to

$$\hat{P}_* F + F^T \hat{P}_* - \hat{P}_* G R^{-1} G^T \hat{P}_* + Q_M = 0 \tag{5.23}$$

The linear part of the optimal control for the nonlinear system thus becomes

$$\underline{u}_*^{(1)}(\underline{z}) = \bar{K} \underline{z} = -R^{-1} G^T \hat{P}_* \underline{z} \tag{5.24}$$

and

$$J^{(2)}(\underline{z}) = \underline{z}(o)^T \hat{P}_* \underline{z}(o) \tag{5.25}$$

At this stage the eigenvalues of the closed loop matrix $F_* = (F + G \bar{K})$ are computed and the stabilizability aspect of the system checked up. The next step is to find the second and the higher degree terms for the feedback control and performance index given by

$$\underline{u}_*(\underline{z}) = \underline{u}_*^{(1)}(\underline{z}) + \underline{u}_*^{(2)}(\underline{z}) + \ldots \tag{5.26}$$

$$J_*(\underline{z}) = J^2(\underline{z}) + J^{(3)}(\underline{z}) + \ldots \tag{5.27}$$

As fairly accurate results can be obtained by approximating the controller in (5.26) as the sum of the first two terms, the computations can be stopped with $\underline{u}_*^{(2)}(\underline{z})$ and $J^{(3)}(\underline{z})$.

The equation corresponding to (4.30) reduces in this case to

$$F_* \underline{z}.J_z^{(3)}(\underline{z}) = g^{(2)}(\underline{z}).J_z^{(2)}(\underline{z}) \tag{5.28}$$

In (5.28) $\underline{g}^{(2)}(\underline{z}) = \underline{g}(\underline{z})$ because care was taken in Section 4.4 to incorporate any linear terms in the F matrix, and thus only second degree terms are left in $\underline{g}(\underline{z})$. Also

$$J_z^{(2)}(\underline{z}) = \begin{bmatrix} 2p_{11}z_1 + 2p_{12}z_2 + 2p_{13}z_3 + 2p_{14}z_4 + 2p_{15}z_5 \\ 2p_{22}z_2 + 2p_{12}z_1 + 2p_{23}z_3 + 2p_{24}z_4 + 2p_{25}z_5 \\ 2p_{33}z_3 + 2p_{13}z_1 + 2p_{23}z_2 + 2p_{34}z_4 + 2p_{35}z_5 \\ 2p_{44}z_4 + 2p_{14}z_1 + 2p_{24}z_2 + 2p_{34}z_3 + 2p_{45}z_5 \\ 2p_{55}z_5 + 2p_{15}z_1 + 2p_{25}z_2 + 2p_{35}z_3 + 2p_{45}z_4 \end{bmatrix} \quad (5.29)$$

where p_{ij}'s are the elements of the solution matrix $\hat{P}_*$ of (5.23) and

$$F_* = (F + G\,\overline{K}) \quad (5.30)$$

The only unknown term is $J_z^{(3)}(\underline{z})$ which is a vector valued function having the partial derivatives of $J^{(3)}(\underline{z})$ with respect to z_1 to z_5 respectively as its elements. Hence $J^{(3)}(\underline{z})$ is assumed to take the following third degree form:

$$\begin{aligned} J^{(3)}(\underline{z}) = {} & d_1z_1^3 + d_2z_1^2z_2 + d_3z_1^2z_3 + d_4z_1^2z_4 + d_5z_1^2z_5 + d_6z_1z_2^2 \\ & + d_7z_1z_3^2 + d_8z_1z_4^2 + d_9z_1z_5^2 + d_{10}z_2^3 + d_{11}z_2^2z_3 \\ & + d_{12}z_2^2z_4 + d_{13}z_2^2z_5 + d_{14}z_2z_3^2 + d_{15}z_2z_4^2 + d_{16}z_2z_5^2 \\ & + d_{17}z_3^3 + d_{18}z_3^2z_4 + d_{19}z_3^2z_5 + d_{20}z_3z_4^2 + d_{21}z_3z_5^2 \\ & + d_{22}z_4^3 + d_{23}z_4^2z_5 + d_{24}z_4z_5^2 + d_{25}z_5^3 + d_{26}z_1z_2z_3 \\ & + d_{27}z_1z_2z_4 + d_{28}z_1z_2z_5 + d_{29}z_1z_3z_4 + d_{30}z_1z_3z_5 \\ & + d_{31}z_1z_4z_5 + d_{32}z_2z_3z_4 + d_{33}z_2z_3z_5 + d_{34}z_2z_4z_5 \\ & + d_{35}z_3z_4z_5 \end{aligned} \quad (5.31)$$

Substituting for $\underline{g}^{(2)}(\underline{z})$, $J_z^{(2)}(\underline{z})$ from (5.29), E_* from (5.30) and $J_z^{(3)}(\underline{z})$ obtained from (5.31) into (5.28) and equating like powers of $\underline{z}$ on both sides, the equation ultimately reduces to the solution of simultaneous equations in 35 unknowns. The same is written as

$$\hat{\Lambda}\,\underline{m} = \hat{\underline{\varepsilon}} \tag{5.32}$$

Here $\hat{\Lambda}$ is a (35x35) coefficient matrix, $\underline{m}$ is a 35 element vector and $\hat{\underline{\varepsilon}}$ is the right hand vector which is known. The description for the matrix $\hat{\Lambda}$ and that for the vector $\hat{\varepsilon}$ are given in Appendix E. After determining the solution vector $\underline{m}$ of (5.32) the second degree control function $\underline{u}^{(2)}(\underline{z})$ is computed. Here $\underline{u}^{(2)}(\underline{z})$ is a vector valued function consisting of two elements. With l=2, (4.31) in this case reduces to

$$\underline{u}_*^{(2)}(\underline{z}) = -\frac{1}{2}R^{-1}\;G^T\;J_z^{(3)}(\underline{z}) \tag{5.33}$$

All the quantities on the right hand side of (5.33) are known. The optimal feedback control for the 5th order system is now given as the sum of the right hand expressions in (5.24) and (5.33)

$$\underline{u}_*(\underline{z}) = \underline{u}_*^{(1)}(\underline{z}) + \underline{u}_*^{(2)}(\underline{z}) + \ldots \tag{5.34}$$

or

$$\underline{u}_*(\underline{z}) = -R^{-1}\;G^T\;\hat{P}\;\underline{z} - \frac{1}{2}R^{-1}\;G^T\;J_z^{(3)}(\underline{z}) \tag{5.35}$$

Substituting C $\underline{x}$ for $\underline{z}$ in the right hand side of (5.34) resulting expression, consisting of first and second degree terms of the elememts of the vector $\underline{x}$, becomes a suboptimal control for the original 8th order system.

5.4 REDUCTION TO 5TH ORDER - COMPUTATIONAL RESULTS

The computation starts with assumption of the range of the excursion of the angle $\Delta\delta_{12} = 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt)$. This is achieved by assuming an initial condition for the variables $\int\Delta f_1 dt$ and $\int\Delta f_2 dt$. By applying the above described feedback control, the angle $\Delta\delta_{12}$ starts from the given nonzero initial value and ultimately reaches a zero steady state; and hence the range is taken as from zero to this initial value. For instance one set of initial conditions taken for the state variables is

$$x_1(o) = \frac{0.30}{2\pi} \quad ; \quad x_5(o) = \frac{0.01}{2\pi}$$

with the rest of the state variables of the 8th order system assumed to be zero. Thus the initial angular perturbation $\Delta\delta_{12} = 2\pi(\int\Delta f_1 dt - \int\Delta f_2 dt) = 0.29$ and the range of $\Delta\delta_{12}$ is from zero to 0.29 radian. The next step is the computation of the H matrix of (4.40). To accomplish this the values of the scalar functions $\theta_1(\underline{x})$ and $\theta_2(\underline{x})$ corresponding to 0.145 radian (= 0.29/2) are obtained by making use of Figure 4.1. Using these values for $\theta_1(\underline{x})$ and $\theta_2(\underline{x})$ the H matrix (8x8) is computed as

$$\begin{bmatrix}
0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-3.123 & -0.050 & 6.000 & 0.0 & 3.123 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -3.333 & 3.333 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -5.208 & 0.0 & -12.500 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\
3.123 & 0.0 & 0.0 & 0.0 & -3.123 & -0.05 & 6.000 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.333 & 3.333 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -5.208 & 0.0 & -12.500
\end{bmatrix}$$

The eigenvalues of the matrix H are computed as

1) $-0.15883\times10^{1} + j0.0$
2) $-0.24247\times10^{-12} + j0.0$
3) $-0.13290\times10^{2} + j0.0$
4) $-0.13266\times10^{2} + j0.0$
5) $-0.12966\times10^{1} + j0.25127\times10^{1}$
6) $-0.51441\times10^{0} + j0.34764\times10^{1}$
7) $-0.12966\times10^{1} - j0.25127\times10^{1}$
8) $-0.51441\times10^{0} - j0.34764\times10^{1}$

The five dominant eigenvalues bearing numbers 2), 6), 8), 5), 7) in the above eigenvalues are considered and the corresponding eigenvectors are computed. Finally the 5x8 matrix C the rows of which are constituted of these eigenvectors, is computed as

$$\begin{bmatrix} 0.544 & 0.213 & 0.384 & 0.102 & 0.544 & 0.213 & 0.384 & 0.107 \\ 0.435 & -0.111 & -0.369 & -0.103 & -0.435 & 0.111 & 0.369 & 0.103 \\ 0.265 & 0.264 & 0.107 & 0.0 & -0.265 & -0.264 & -0.107 & 0.0 \\ 0.0 & 0.144 & 0.588 & 0.175 & 0.0 & 0.144 & 0.588 & 0.175 \\ 0.0 & -0.291 & -0.132 & 0.0 & 0.0 & -0.291 & -0.132 & 0.0 \end{bmatrix}$$

The matrices F (5x5) and G (5x2) of (4.50) and the matrix Q_M (5x5) of (4.52) are computed as

$$F = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.48089\times10^{0} & 0.35215\times10^{1} & 0.0 & 0.0 \\ 0.0 & -0.35559\times10^{1} & -0.62135\times10^{0} & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.12966\times10^{1} & 0.25127\times10^{1} \\ 0.0 & 0.0 & 0.0 & -0.25127\times10^{1} & -0.12966\times10^{1} \end{bmatrix}$$

$$G = \begin{bmatrix} 0.12802\text{x}10^{1} & -0.12832\text{x}10^{1} & 0.0 & 0.21884\text{x}10^{1} & 0.0 \\ 0.12802\text{x}10^{1} & 0.12832\text{x}10^{1} & 0.0 & 0.21884\text{x}10^{1} & 0.0 \end{bmatrix}^{T}$$

$$Q_M = \begin{bmatrix} 0.393\text{x}10^{-5} & 0.0 & 0.0 & -0.107\text{x}10^{-2} & -0.525\text{x}10^{-2} \\ 0.0 & 0.165\text{x}10^{0} & -0.552\text{x}10^{0} & 0.0 & 0.0 \\ 0.0 & -0.552\text{x}10^{0} & 0.185\text{x}10^{1} & 0.0 & 0.0 \\ -0.107\text{x}10^{-2} & 0.0 & 0.0 & 0.293\text{x}10^{0} & 0.143\text{x}10^{1} \\ -0.524\text{x}10^{-2} & 0.0 & 0.0 & 0.143\text{x}10^{1} & 0.700\text{x}10^{1} \end{bmatrix}$$

The next step is determination of the feedback controller consisting of first degree and second degree terms in the elements of the vector $\underline{z}$.

The truncated control problem described by (5.22) and (5.21) is then solved. At this stage the controllability of the (F,G) pair is checked up. Next, taking R as 2x2 identity matrix, the solution matrix $\hat{P}_*$ of (5.23) is computed as 5x5 matrix

$$\begin{bmatrix} 0.229\text{x}10^{-3} & 0.0 & 0.0 & 0.507\text{x}10^{-3} & -0.151\text{x}10^{-2} \\ 0.0 & 0.509\text{x}10^{0} & -0.165\text{x}10^{0} & 0.0 & 0.0 \\ 0.0 & -0.165\text{x}10^{0} & 0.478\text{x}10^{0} & 0.0 & 0.0 \\ 0.507\text{x}10^{-3} & 0.0 & 0.0 & 0.335\text{x}10^{0} & -0.328\text{x}10^{0} \\ -0.151\text{x}10^{-2} & 0.0 & 0.0 & -0.328\text{x}10^{0} & 0.166\text{x}10^{1} \end{bmatrix}$$

The linear feedback matrix $\overline{K}$ (2x5) of (5.24) is computed as

$$\begin{bmatrix} -0.140\text{x}10^{-2} & 0.653\text{x}10^{0} & -0.212\text{x}10^{0} & -0.733\text{x}10^{0} & 0.720\text{x}10^{0} \\ -0.140\text{x}10^{-2} & -0.653\text{x}10^{0} & 0.212\text{x}10^{0} & -0.733\text{x}10^{0} & 0.720\text{x}10^{0} \end{bmatrix}$$

The matrix F_* of (5.30) is computed as

$$\begin{bmatrix} -0.359\times10^{-2} & 0.0 & 0.0 & -0.188\times10^{1} & 0.184\times10^{1} \\ 0.0 & -0.216\times10^{1} & 0.407\times10^{1} & 0.0 & 0.0 \\ 0.0 & -0.356\times10^{1} & -0.621\times10^{0} & 0.0 & 0.0 \\ -0.614\times10^{-2} & 0.0 & 0.0 & -0.450\times10^{1} & 0.567\times10^{1} \\ 0.0 & 0.0 & 0.0 & -0.251\times10^{1} & -0.130\times10^{1} \end{bmatrix}$$

The stabilizability aspect of the system is now checked up by computing the eigenvalues of F_* as

1) $-0.14294\times10^{-2} + j0.0$
2) $-0.29017\times10^{1} + j0.34142\times10^{1}$
3) $-0.29017\times10^{1} - j0.34142\times10^{1}$
4) $-0.13885\times10^{1} + j0.37243\times10^{1}$
5) $-0.13885\times10^{1} - j0.37243\times10^{1}$

As all the eigenvalues are having negative real parts, the 5th order system is stabilizable.

The coefficient matrix $\hat{\Lambda}$ and the right hand vector $\underline{\hat{\varepsilon}}$ of (5.32) are computed and given in Appendix E. The elements of the solution vector $\underline{m}$ (35x1) are computed as

$$m_{10} = -0.44360\times10^{0}$$
$$m_{11} = 0.56631\times10^{-1}$$
$$m_{12} = -0.12688\times10^{1}$$
$$m_{17} = 0.96424\times10^{0}$$

The rest of the elements being zero.

$J^{(3)}(\underline{z})$ as well as $J_z^{(3)}(\underline{z})$ are now completely determined. The 2-element vector feedback function $\underline{u}_*^{(2)}(\underline{z})$ is finally determined as

$$\begin{bmatrix} 0.53455z_2^2 + 0.07267z_2z_3 - 1.6281z_2z_4 \\ 2.2422z_2^2 - 0.07267z_2z_3 + 1.6281z_2z_4 \end{bmatrix}$$

Now the optimal control vector function $u_*(z)$ is given as

$$\underline{u}_*(\underline{z}) = \underline{u}_*^{(1)}(\underline{z}) + \underline{u}_*^{(2)}(\underline{z})$$

where

$$\underline{u}_*^{(1)}(z) = \bar{K}\ \underline{z}$$

as given in (5.24) and $\underline{u}_*^{(2)}(\underline{z})$ is computed as above. By substituting $\underline{z} = C\ \underline{x}$ in the above optimal control vector, the same will become the suboptimal control vector say $\underline{v}_*(\underline{x})$ (2x1) for the 8th order LFC system.

The responses of the state variables Δf_1 and Δf_2 of the 8th order LFC system with $\underline{v}_*(\underline{x})$ as a feedback control function are determined and are given as continuous curves in Figure 5.3. For comparison, the system is linearized about the operating point, the linear suboptimal control function determined by 5th order aggregation in the same lines as detailed in Chapter 3 and applied to this nonlinear system. The same is given as dashed curve in Figure 5.3. Two other initial conditions are taken, (a) $x_1 = 0.06/2\pi$, $x_5 = 0.001/2\pi$, rest of the states zero and (b) $x_1 = 1.099/2\pi$, $x_5 = 0.0174/2\pi$, rest of the states zero; and the suboptimal

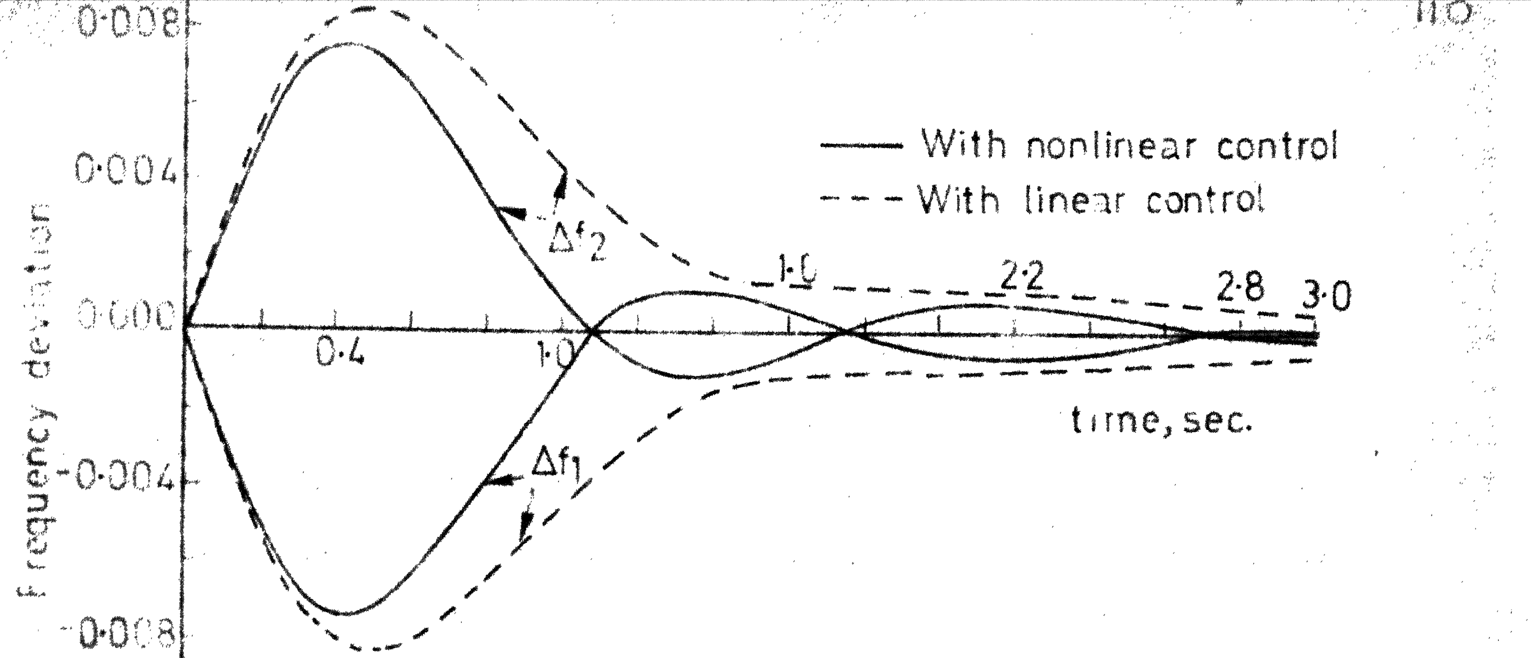


Fig. 5·2 Suboptimal responses of two-area system (reduction 8 to 5) with initial condition x_1-0·06/2π, x_5-0·001/2π, rest zero

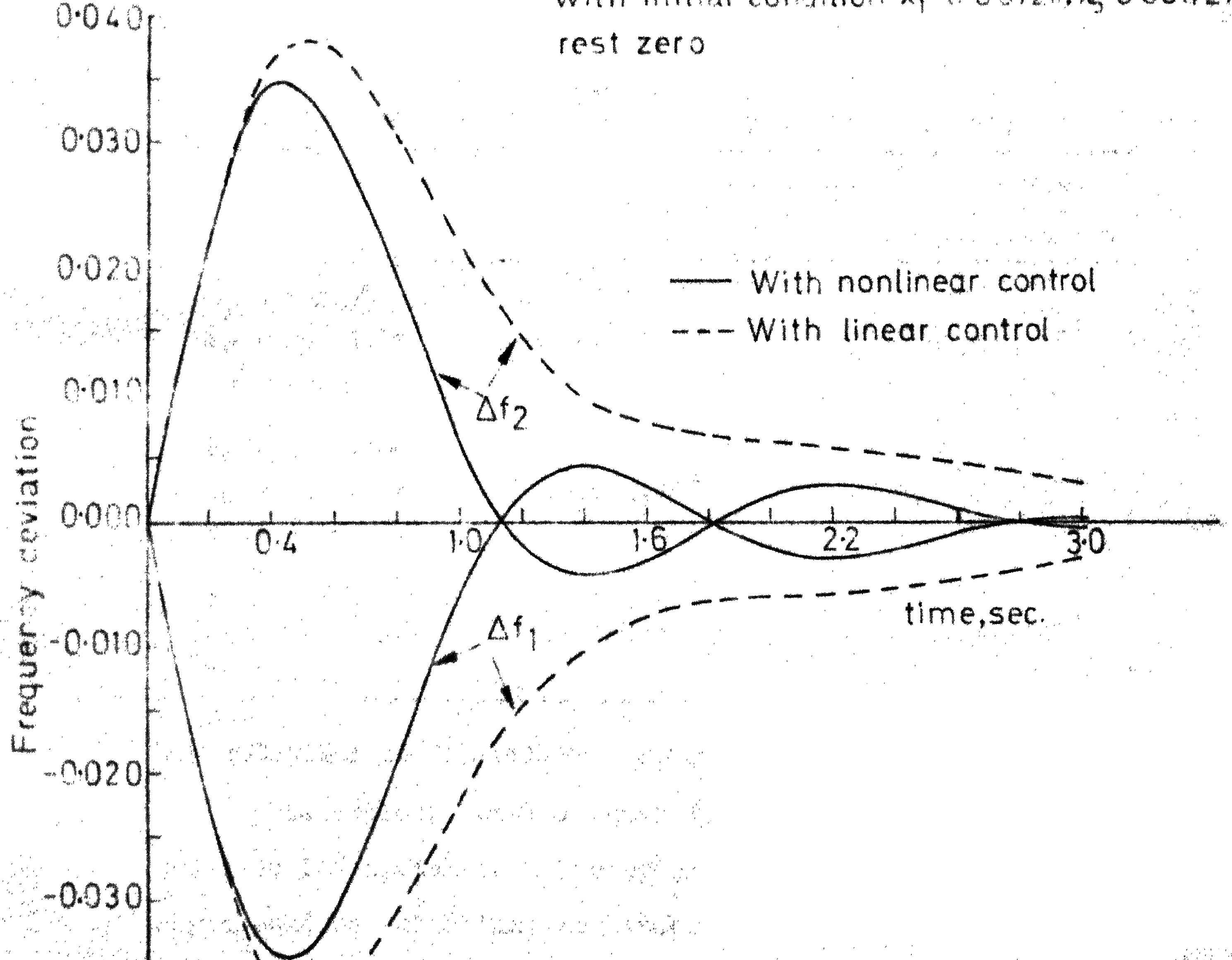


Fig. 5·3 Suboptimal responses of two-area system (reduction 8to5) with initial condition x_1=0·30/2π, x_5=0·01/2π, rest zero

responses of the state variables Δf_1 and Δf_2 are determined as shown in Figures 5.2 and 5.4 respectively. Also the responses obtained by applying the suboptimal control assuming the system to be linear are shown as dashed curves in Figures 5.2 and 5.4 respectively. The performance index figures $J^{(2)}(\underline{x})$ obtained with the suboptimal nonlinear control and suboptimal linear control and determined for the three initial conditions considered above are given in Table 5.1.

Table 5.1

Reduction to 5th order - performance indices.

Initial condition	$J^{(2)}(\underline{x})$ with nonlinear suboptimal control	$J^{(2)}(\underline{x})$ with linear suboptimal control
$x_1 = 0.06/2\pi$ $x_5 = 0.001/2\pi$ rest zero	0.59226×10^{-4}	0.95747×10^{-4}
$x_1 = 0.30/2\pi$ $x_5 = 0.01/2\pi$ rest zero	0.13136×10^{-2}	0.21120×10^{-2}
$x_1 = 1.099/2\pi$ $x_5 = 0.0174/2\pi$ rest zero	0.10183×10^{-1}	0.17237×10^{-1}

5.5 REDUCTION TO 3RD ORDER - COMPUTATIONAL RESULTS

The computational results for the reduction of the two-area LFC system to a 3rd order model and for the corresponding suboptimal nonlinear regulation, are given

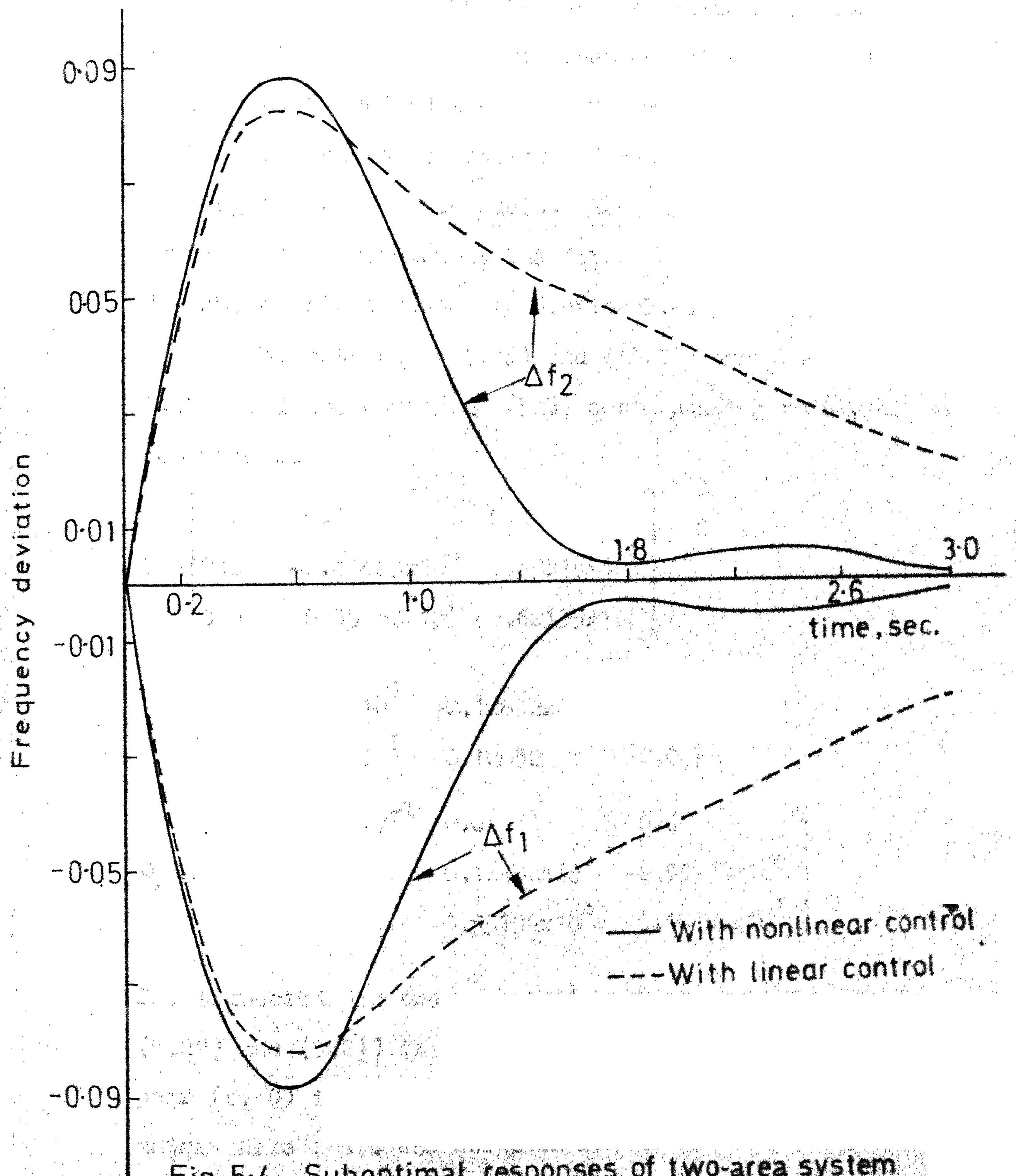


Fig. 5·4 Suboptimal responses of two-area system (reduction 8 to 5) with initial condition $x_1 = 1{\cdot}099/2\pi$, $x_5 = 0{\cdot}0174/2\pi$, rest ze

in this section. The matrix H (see (4.40)) remains the same as computed in Section 5.4. The three dominant eigenvalues out of the eight ones computed are bearing numbers 2),6) and 8) therein. The rows of the matrix C (3x8) in this case consist of the eigenvectors corresponding to these eigenvalues. Thus the three rows of the matrix C are computed as the first three rows of the C (5x8) matrix computed in Section 5.4.

The matrices F (3x3) and G(3x2) corresponding to (4.50) and the matrix Q_M (3x3) corresponding to (4.52) are computed as

$$F = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & -0.48089\times10^{0} & 0.35215\times10^{1} \\ 0.0 & -0.35559\times10^{1} & -0.62135\times10^{0} \end{bmatrix}$$

$$G = \begin{bmatrix} 0.12802\times10^{1} & -0.12832\times10^{1} & 0.0 \\ 0.12802\times10^{1} & 0.12832\times10^{1} & 0.0 \end{bmatrix}^T$$

$$Q_M = \begin{bmatrix} 0.91212\times10^{-1} & 0.0 & 0.0 \\ 0.0 & 0.16488\times10^{0} & -0.55199\times10^{0} \\ 0.0 & -0.55199\times10^{0} & 0.18480\times10^{1} \end{bmatrix}$$

The truncated 3rd order control problem corresponding to (5.22) and (5.21) is solved. The controllability of the pair (F, G) is checked up. The matrix R is a 2x2 identity matrix here also. The solution matrix $\hat{P}_*$ of the 3rd order equation corresponding to (5.23) is computed as the

3x3 matrix

$$\begin{bmatrix} 0.16681 & 0.0 & 0.0 \\ 0.0 & 0.50855 & -0.16534 \\ 0.0 & -0.16534 & 0.47753 \end{bmatrix}$$

The linear feedback matrix $\bar{K}$ (2x3) of (5.24) and the matrix F_* of (5.30) are computed as

$$\bar{K} = \begin{bmatrix} -0.21356 & 0.65256 & -0.21216 \\ -0.21356 & -0.65256 & 0.21216 \end{bmatrix}$$

$$F_* = \begin{bmatrix} -0.54681\text{x}10^0 & 0.0 & 0.0 \\ 0.0 & -0.21556\text{x}10^1 & 0.40660\text{x}10^1 \\ 0.0 & -0.35559\text{x}10^1 & -0.62135\text{x}10^0 \end{bmatrix}$$

The stabilizability of the system is checked up by computing the eigenvalues of F_* as

1) $-0.54681\text{x}10^0 + j0.0$
2) $-0.55112\text{x}10^0 + j0.25617\text{x}10^1$
3) $-0.55112\text{x}10^0 - j0.25617\text{x}10^1$

As all the eigenvalues are having negative real parts, the system is stabilizable.

The linear part of the controller is given as $\underline{u}_*^{(1)}(\underline{z}) = \bar{K}\,\underline{z}$ and the closed loop system incorporating this control is given as $\dot{\underline{z}} = F_*\,\underline{z}$.

The equation corresponding to (4.30) reduces in this case also to:

$$F_*\,\underline{z}.J_z^{(3)}(\underline{z}) = \underline{g}^{(2)}(\underline{z}).J_z^{(2)}(\underline{z}) \qquad (5.36)$$

Here F_* is computed as above. $g^{(2)}(\underline{z}) = g(\underline{z})$ as before and consists of only second degree terms in the elements of $\underline{z}$.

$$J_z^{(2)}(\underline{z}) = \begin{bmatrix} 2p_{11}z_1 + 2p_{12}z_2 + 2p_{13}z_3 \\ 2p_{22}z_2 + 2p_{12}z_1 + 2p_{23}z_3 \\ 2p_{33}z_3 + 2p_{13}z_1 + 2p_{23}z_2 \end{bmatrix} \tag{5.37}$$

where p_{ij}'s are the elements of the matrix $\hat{P}_*$ computed as above. The only unknown term is $J_z^{(3)}(\underline{z})$ and corresponding to this, $J^{(3)}(\underline{z})$ is assumed as

$$J^{(3)}(\underline{z}) = l_1z_1^3 + l_2z_2^3 + l_3z_3^3 + l_4z_1^2z_2 + l_5z_1z_2^2 + l_6z_1^2z_3 + l_7z_1z_3^2 + l_8z_2^2z_3 + l_9z_2z_3^2 + l_{10}z_1z_2z_3 \quad .. \tag{5.38}$$

$$J_z^{(3)}(\underline{z}) = \begin{bmatrix} 3l_1z_1^2 + 2l_4z_1z_2 + l_5z_2^2 + 2l_6z_1z_3 + l_7z_3^2 + l_{10}z_2z_3 \\ 3l_2z_2^2 + l_4z_1^2 + 2l_5z_1z_2 + 2l_8z_2z_3 + l_9z_3^2 + l_{10}z_1z_3 \\ 3l_3z_3^2 + l_6z_1^2 + 2l_7z_1z_3 + l_8z_2^2 + 2l_9z_2z_3 + l_{10}z_1z_2 \end{bmatrix} \quad .. \tag{5.39}$$

Substituting the above terms in (5.36) and equating like powers of $\underline{z}$, the problem reduces to the solution of simultaneous equations in 10 unknowns as given by

$$\Lambda \, \underline{m} = \underline{\varepsilon} \tag{5.40}$$

Here the description of the 10x10 matrix Λ is the same as that in (4.86). However, f_{ij}'s are the elements of F_* matrix computed in this section as above. The elements $\varepsilon_1, \ldots, \varepsilon_{10}$ of the right hand vector $\underline{\varepsilon}$ are described below. Let

$$q_1 = \frac{(0.5f^* \tau_{12} \pi^2)}{H_1} (c_{12} - c_{16})$$

$$q_2 = \frac{(0.5f^* \tau_{12} \pi^2)}{H_1} (c_{22} - c_{26})$$

$$q_3 = \frac{(0.5f^* \tau_{12} \pi^2)}{H_1} (c_{32} - c_{36})$$

where c_{ij}'s are the elements of the C matrix. Also let

$$s_1 = t_{11} - t_{51}, \quad s_2 = t_{12} - t_{52}, \quad s_3 = t_{13} - t_{53}$$

where t_{ij}'s are the elements of the (8x3) matrix $\nabla = C^T(CC^T)^{-1}$

Then

$$\varepsilon_1 = 2s_1^2(q_1 p_{11} + q_2 p_{12} + q_3 p_{13})$$

$$\varepsilon_2 = 2s_2^2(q_1 p_{12} + q_2 p_{22} + q_3 p_{23})$$

$$\varepsilon_3 = 2s_3^2(q_1 p_{13} + q_2 p_{23} + q_3 p_{33})$$

Now denoting

$$y_1 = q_1 p_{11} + q_2 p_{12} + q_3 p_{13}$$

$$y_2 = q_1 p_{12} + q_2 p_{22} + q_3 p_{23}$$

and

$$y_3 = q_1 p_{13} + q_2 p_{23} + q_3 p_{33},$$ we have

$$\varepsilon_4 = 4s_1 s_2 y_1 + 2s_1^2 y_2$$

$$\varepsilon_5 = 2s_2^2 y_1 + 4s_1 s_2 y_2$$

$$\varepsilon_6 = 4s_1 s_2 y_1 + 2s_1^2 y_3$$

$$\varepsilon_7 = 2s_3^2 y_1 + 4s_1 s_3 y_3$$

$$\varepsilon_8 = 4s_2 s_3 y_2 + 2s_2^2 y_3$$

$$\varepsilon_9 = 2s_3^2 y_2 + 4s_2 s_3 y_3$$

$$\varepsilon_{10} = 4(s_2 s_3 y_1 + s_1 s_3 y_2 + s_1 s_2 y_3)$$

.. (5.42)

Here the coefficient matrix Λ is computed as given in (5.41). The elements of the right hand vector $\underline{\varepsilon}$ of (5.40) are computed as

$$\varepsilon_2 = 0.26673\text{x}10^1 , \quad \varepsilon_3 = -0.69565\text{x}10^1 ,$$

$$\varepsilon_8 = 0.33333\text{x}10^1 , \quad \varepsilon_9 = -0.55140\text{x}10^1$$

the rest of the elements being zero. Also the elements of the solution vector, $\underline{m}$, are computed as

$$m_2 = -0.52931\text{x}10^0 , \quad m_3 = 0.10679\text{x}10^1 ,$$

$$m_8 = 0.21249\text{x}10^0 , \quad m_9 = -0.12213\text{x}10^1$$

the rest of the elements being zero. Thus the expression for $J_z^{(3)}(\underline{z})$ given in (5.39) is now determined fully. Finally the expression for $\underline{u}_*^{(2)}(\underline{z})$ corresponding to (4.31) with l=2, is written as

$$\underline{u}_*^{(2)}(\underline{z}) = -\tfrac{1}{2}R^{-1}G^T J_z^{(3)}(\underline{z}) \qquad (5.43)$$

and is determined as

$$\underline{u}_*^{(2)}(\underline{z}) \quad \begin{bmatrix} -1.0188z_2^2 - 0.7836z_3^2 + 0.2727z_2z_3 \\ 1.0188z_2^2 + 0.7836z_3^2 - 0.2727z_2z_3 \end{bmatrix} \tag{5.44}$$

As before, the optimal control vector $\underline{u}_*(\underline{z})$ (2x1) for the reduced system is given as

$$\underline{u}_*(\underline{z}) = \underline{u}_*^{(1)}(\underline{z}) + \underline{u}_*^{(2)}(\underline{z}) \tag{5.45}$$

where $\underline{u}_*^{(1)}(\underline{z})$ and $\underline{u}_*^{(2)}(\underline{z})$ are computed as above. By substituting $\underline{z} = C\,\underline{x}$ in the expression for the above vector function, the resulting vector function say $\underline{v}\,(\underline{x})$ (2x1) becomes a suboptimal feedback control for the original 8th order system.

The suboptimal responses of the two state variables Δf_1 and Δf_2 of the 8th order system with each of the three initial conditions considered in Section 5.4 are given as continuous curves in Figures 5.5 to 5.7. For the same initial conditions, the responses obtained with linear suboptimal control determined by linearizing the 8th order system about the operating point and using the 3rd order aggregation are given as dashed curves in Figures 5.5 to 5.7.

As in the 5th order case, the performance index figures $J^{(2)}(\underline{x})$ with the above two kinds of suboptimal control are computed and given in Table 5.2.

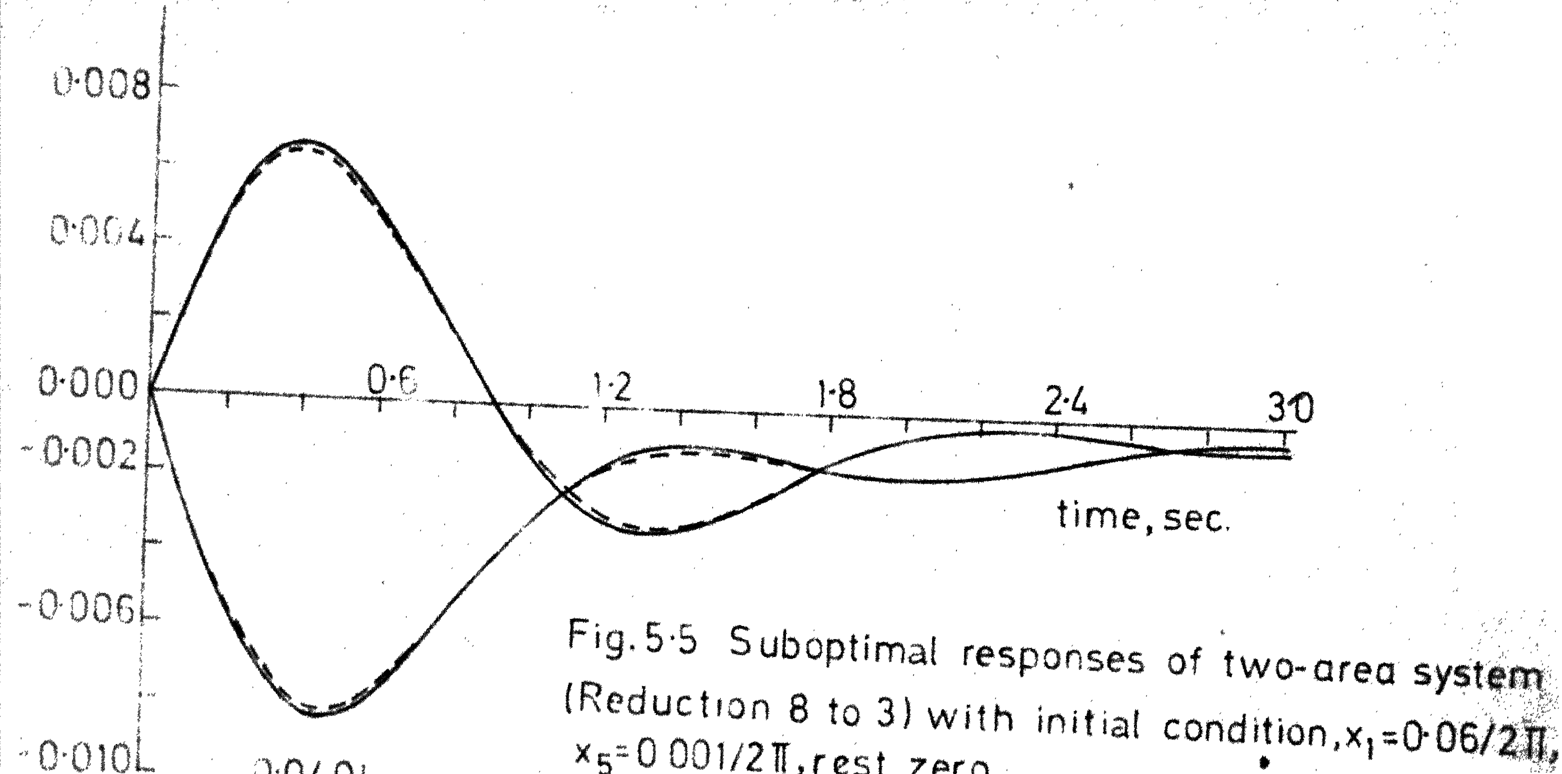


Fig. 5·5 Suboptimal responses of two-area system (Reduction 8 to 3) with initial condition, $x_1 = 0{\cdot}06/2\pi$, $x_5 = 0{\cdot}001/2\pi$, rest zero

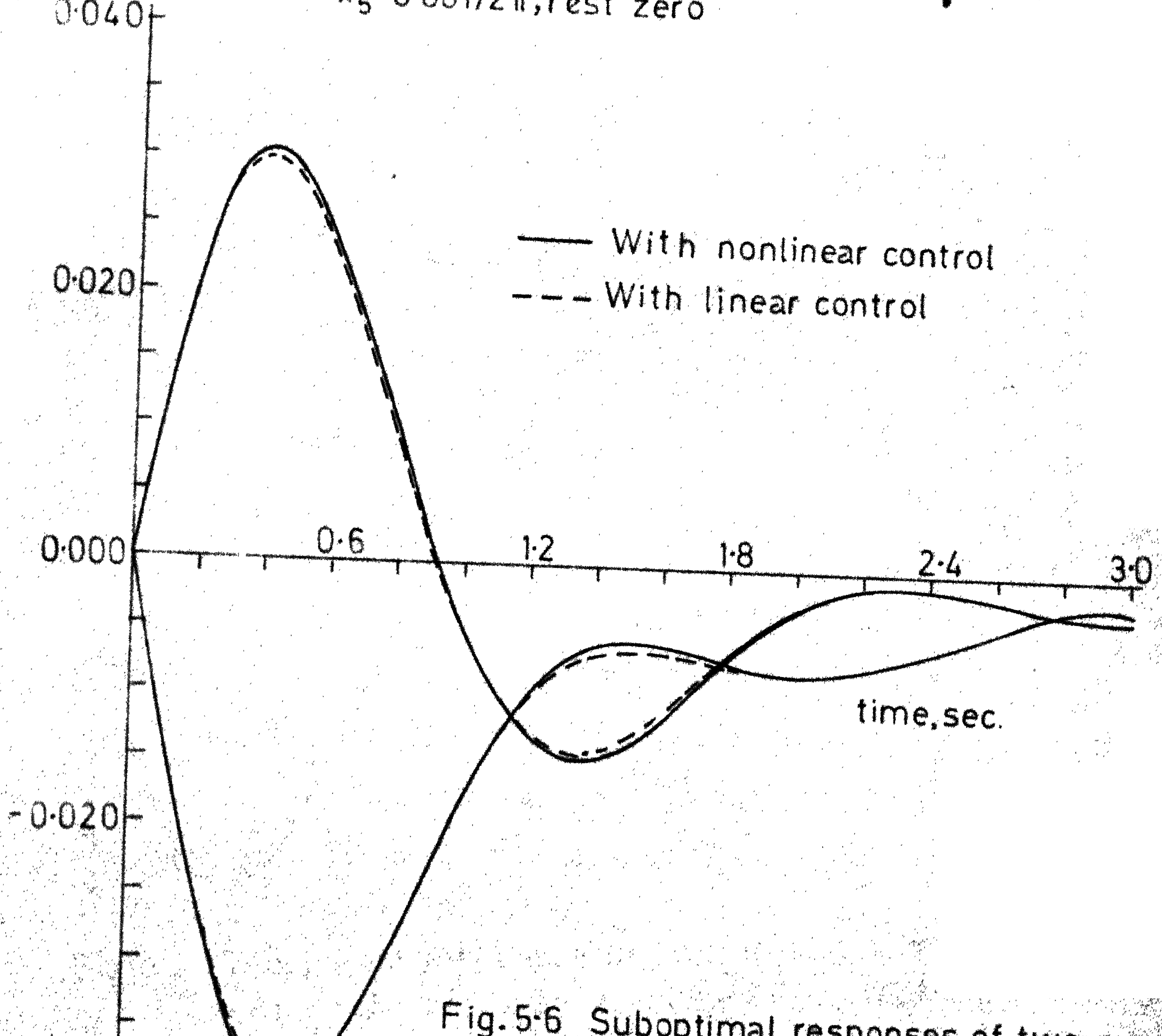


Fig. 5·6 Suboptimal responses of two-area system (Reduction 8 to 3) with initial condition $x_1 = 0{\cdot}30/2\pi$, $x_5 = 0{\cdot}01/2\pi$

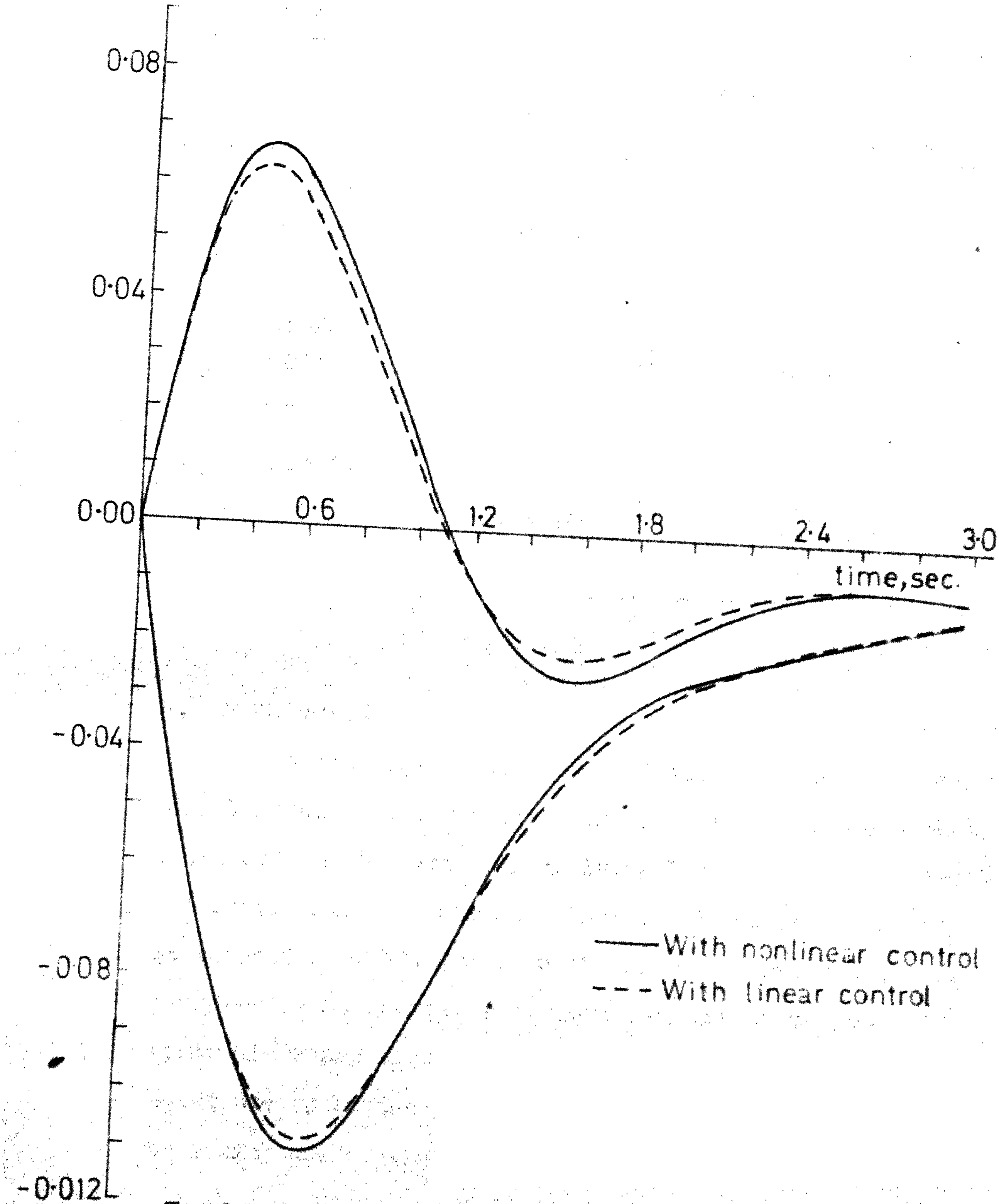


Fig. 5·7 Suboptimal responses of two-area system (reduction 8 to 3) with initial condition $x_1 = 1{\cdot}099/2\pi$, $x_5 = 0{\cdot}0174/2\pi$ rest zero

Table 5.2
Reduction to 3rd order - performance indices.

Initial condition	$J^{(2)}(\underline{x})$ with suboptimal nonlinear control	$J^{(2)}(\underline{x})$ with suboptimal linear control
$x_1 = 0.06/2\pi$ $x_5 = 0.001/2\pi$ rest zero	0.74985×10^{-4}	0.66922×10^{-4}
$x_1 = 0.30/2\pi$ $x_5 = 0.01/2\pi$ rest zero	0.16964×10^{-2}	0.15166×10^{-2}
$x_1 = 1.099/2\pi$ $x_5 = 0.0174/2\pi$ rest zero	0.15415×10^{-1}	0.13659×10^{-1}

5.6 CONCLUSIONS

A suboptimal controller is determined for the large signal model of the two-area system by reducing the same to a 5th order model and then applying Lukes' method for optimal regulation to the reduced model. Secondly the system is reduced to a 3rd order model and suboptimal controller constructed as above. For comparison linear suboptimal controllers are also determined by linearizing the system about the operating point and reducing this linear model by making use of the method of aggregation due to Aoki[6].

From Figures 5.2 to 5.4 it is seen that the sub-optimal responses obtained by the method presented are superior to the linear suboptimal controllers. Higher values for performance incides obtained with linear controllers (Table 5.1) also confirm this fact.

From Figures 5.5 to 5.7 and also Table 5.2 it is seen that no advantage is gained by model reduction to 3rd order and applying Lukes' method to the same. This not only confirms the fact that accuracy of representation increases with the order of the reduced model but also shows that for the case of two-area LFC system considered here, the appropriate model for reduction is that of order 5.

Optimal control of nonlinear dynamical systems had been hitherto accomplished by the well known method of linearization of the system at every step. This involves considerable amount of computation; moreover in many cases, the results obtained may not be accurate. Due to the reasons stated above, the method for suboptimal control presented in this chapter is quite promising for systems having trigonometrical sine type of nonlinearity.

CHAPTER 6

DAMPING EFFECTS OF EXCITATION CONTROL IN A LOAD FREQUENCY CONTROL SYSTEM

In the previous chapters the LFC system is studied with the assumption that there is no interaction between the Megawatt frequency and Megavar voltage control loops or equivalently assuming that the excitation system has no effect on the dynamics of the LFC system. This assumption is permissible only when the speed of the excitation system is much faster than the LFC loop; but in practical systems this is not true. The importance of damping due to excitation control on the stability of synchronous machines has been recently realized[27,32] and the same has been studied both by simulation and by conducting actual field tests. Durick[34], is probably the first to investigate the damping effects of voltage control on LFC systems. He studied the effect of area voltage on the tieline power flow for a two-area LFC system; the assumptions made in his study are: (i) the area voltage is available as a control variable which can be manipulated for obtaining optimal response; (ii) that the area voltage perturbation does not have any effect on the area load.

Assumption (i) made by Durick is not true because voltages are controlled in actual systems by means of voltage regulator and exciter loops consisting of the amplifier, exciter, stabilizer and the field time constant. Assumption (ii) again is not correct because it amounts to neglecting an important interaction in LFC systems.

In this chapter a two-area LFC system is studied without making the above assumptions.

Section 6.2 gives a brief description of the system studied by Durick. In Sections 6.3 to 6.5, a more realistic model is treated by including the exciter loop as detailed above; also Section 6.6 gives the computational results for this model.

6.2 OPTIMALLY VOLTAGE DAMPED TWO-AREA SYSTEM[10,34]

The block diagram of the two-area LFC system with voltage perturbations $\Delta |V_1|$ and $\Delta |V_2|$ as additional inputs is given in Figure 6.1. The system differential equations for this system remain the same as given in Appendix A except those for Δf_1 and Δf_2; moreover the state variable $\int \Delta P_{tie1} dt$ has not been considered in the present model. Considering a load disturbance ΔP_{d1} in Area 1 only, the differential equations in respect of Δf_1 and Δf_2 are as follows.

$$\frac{d(\Delta f_1)}{dt} = -\frac{f^* D_1}{2H_1}\Delta f_1 + \frac{f^*}{2H_1}\Delta P_{g1} - \frac{f^*}{2H_1}P_{d1} - \frac{f^*}{2H_1}\Delta P_{tie1} \quad .. \quad (6.1)$$

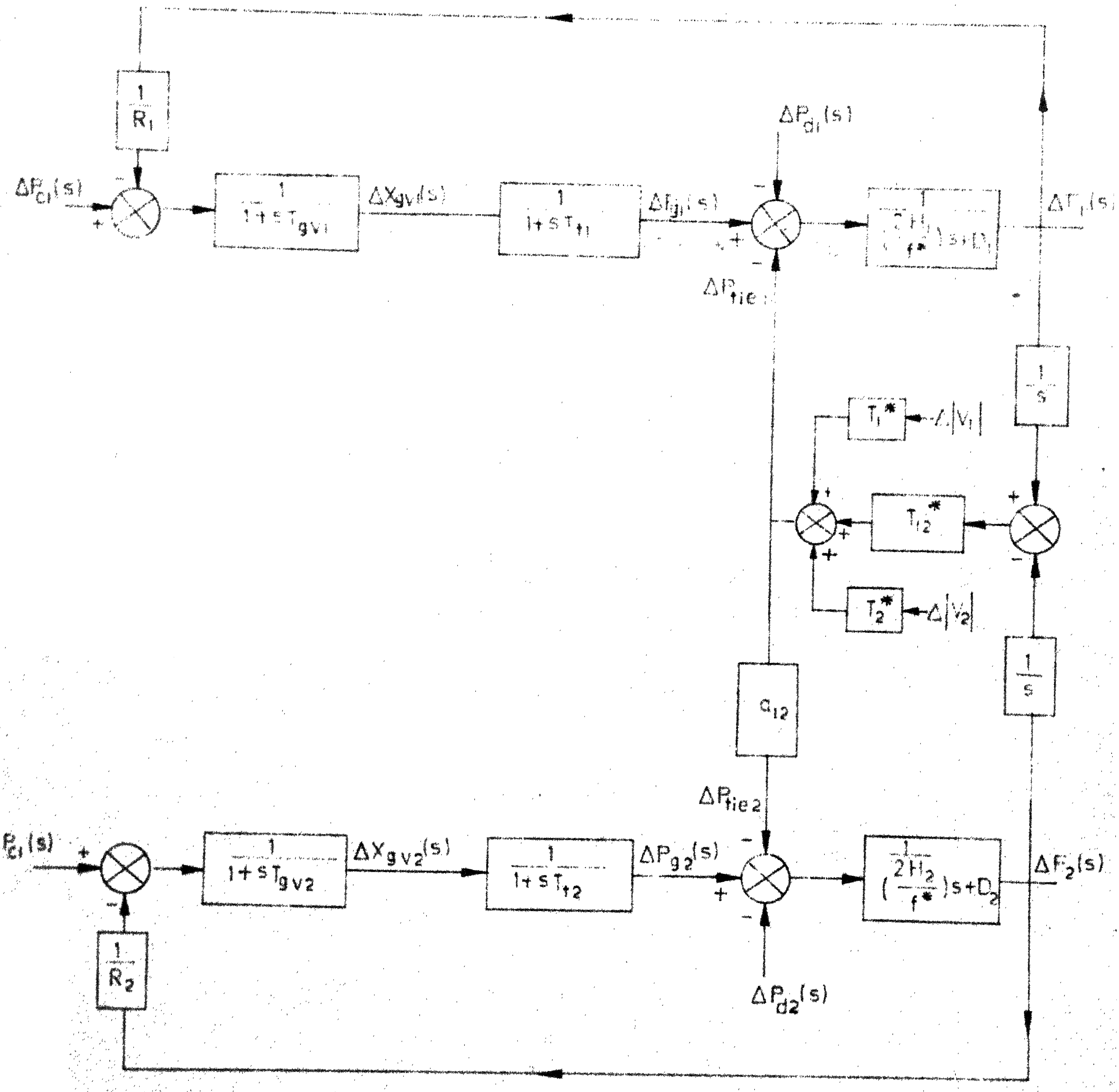


Fig. 6·1 Block diagram of a two-area system with voltage perturbations $\Delta|V_1|$ and $\Delta|V_2|$ added as extra inputs.

$$\frac{d(\Delta f_2)}{dt} = - \frac{f^* D_2}{2H_2}\Delta f_2 + \frac{f^*}{2H_2}\Delta P_{g2} - \frac{f^*}{2H_2}\Delta P_{tie2} \qquad (6.2)$$

The expression for ΔP_{tie1}, assumed positive in the direction 1 to 2 is

$$P_{tie1} = \frac{|V_1|\,|V_2|}{P_{r1}X_{12}} \operatorname{Sin}(\delta_1 - \delta_2) \qquad (6.3)$$

If we assume that the four quantities δ_1, δ_2, $|V_1|$ and $|V_2|$ change by the amounts $\Delta\delta_1$, $\Delta\delta_2$, $\Delta|V_1|$ and $\Delta|V_2|$ respectively, the formula for ΔP_{tie1} becomes

$$\Delta P_{tie1} = \frac{\partial P_{tie1}}{\partial|V_1|}\Delta|V_1| + \frac{\partial P_{tie1}}{\partial|V_2|}|\Delta V_2| + \frac{\partial P_{tie1}}{\partial(\delta_1-\delta_2)}\Delta(\delta_1-\delta_2) \qquad \ldots (6.4)$$

Upon performing the partial differentials called for, and using the relations $\Delta\delta_1 = 2\pi\int\Delta f_1\,dt$ and $\Delta\delta_2 = 2\pi\int\Delta f_2 dt$, (6.4) becomes

$$\Delta P_{tie1} = T_1^*\Delta|V_1| + T_2^*\Delta|V_2| + T_{12}^*(\int\Delta f_1 dt - \int\Delta f_2 dt) \qquad \ldots (6.5)$$

where T_{12}^* is the synchronizing coefficient as defined in Appendix A. The coefficients T_1^* and T_2^* are defined as:

$$T_1^* = \frac{|V_2^*|}{P_{r1}X_{12}} \operatorname{Sin}(\delta_1^* - \delta_2^*) \text{ p.u. MW/p.u.volt} \qquad (6.6)$$

$$T_2^* = \frac{|V_1^*|}{P_{r1}X_{12}} \operatorname{Sin}(\delta_1^* - \delta_2^*) \text{ p.u. MW/p.u.volt} \qquad (6.7)$$

Substituting right hand side of (6.5) for ΔP_{tie1} in (6.1) the latter becomes

$$\frac{d(\Delta f_1)}{dt} = -\frac{f^* T^*_{12}}{2H_1}\int \Delta f_1 dt - \frac{f^* D_1}{2H_1}\Delta f_1 + \frac{f^*}{2H_1}\Delta P_{g1}$$

$$+ \frac{f^* T^*_{12}}{2H_1}\int \Delta f_2 dt - \frac{f^* T^*_1}{2H_1}\Delta |V1| - \frac{f^* T^*_2}{2H_1}\Delta |V_2|$$

$$- \frac{f^*}{2H_1}\Delta P_{d1} \qquad (6.8)$$

Now using the relation

$$\Delta P_{tie2} = a_{12}\Delta P_{tie1} \qquad (6.9)$$

and that in (6.5) a similar differential equation can be written in the case of (6.2) also.

In (A.5) of Appendix A the variables ΔP_{c1} and ΔP_{c2} are the usual LFC system control variables. To study the effect of $\Delta|V_1|$ and $\Delta|V_2|$ only, here the control variables ΔP_{c1} and ΔP_{c2} are considered to be absent. With the above changes the system dynamic equations can be written in the form

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \Gamma\,\Delta\underline{P}_d \qquad (6.10)$$

where $\underline{x}$ is an eight element state vector consisting of the variables $\int\Delta f_1\,dt$, Δf_1, ΔP_{g1}, ΔX_{gv1}, $\int\Delta f_2 dt$, Δf_2, ΔP_{g2} and ΔX_{gv2} as its elements. Also $\underline{u}$ is a 2-element control vector consisting of $\Delta|V_1|$ and $\Delta|V_2|$. Here

$$B = \begin{bmatrix} 0 & -\frac{f^*T^*_1}{2H_1} & 0 & 0 & 0 & -\frac{a_{12}f^*T^*_1}{2H_2} & 0 & 0 \\ 0 & -\frac{f^*T^*_2}{2H_1} & 0 & 0 & 0 & -\frac{a_{12}f^*T^*_2}{2H_2} & 0 & 0 \end{bmatrix}^T \qquad (6.11)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-f^*T^*_{12}}{2H_1} & \frac{-f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 & \frac{f^*T^*_{12}}{2H_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{T_{gv1}R_1} & 0 & \frac{-1}{T_{gv1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{-a_{12}f^*T^*_{12}}{2H_2} & 0 & 0 & 0 & \frac{a_{12}f^*T^*_{12}}{2H_2} & \frac{-f^*D_2}{2H_2} & \frac{f^*}{2H_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{gv2}R_2} & 0 & \frac{-1}{T_{gv2}} \end{bmatrix} \quad (6.12)$$

To apply the optimal control theory to the above system the third term $\Gamma \Delta \underline{P}_d$ in the right hand side of (6.10) is to be eliminated. To accomplish this, the same procedure as was described in Elgerd and Fosha[3] is used. That is, the states and controls have been redefined in terms of their steady state values. Thus the dynamic equation becomes

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \quad (6.13)$$

To pose this as a control problem, the cost function is taken as

$$J = \frac{1}{2}\int_0^\infty (\underline{x}^T\, Q\, \underline{x} + \underline{u}^T\, R\, \underline{u})\, dt \quad (6.14)$$

The matrix Q is chosen such that it gives weightage for the frequency deviations Δf_1 and Δf_2. The same is given as

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (6.15)$$

and R is taken as

$$R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \qquad (6.16)$$

From Pontryagin's minimum principle (Appendix B) the optimal inputs $\Delta |V_1|$ and $\Delta |V_2|$ are computed as follows:

$$\Delta |V_1| = 3.375\text{x}10^{-5}(\int \Delta f_1 dt - \int \Delta f_2 dt) + 4.699 \Delta f_1 + 2.22 \Delta P_{g1} + 0.279 \Delta X_{gv1} - 4.71 \Delta f_2 - 2.24 \Delta P_{g2} - 0.284 \Delta X_{gv2} \qquad (6.17)$$

$$\Delta |V_2| = 3.376\text{x}10^{-5}(\int \Delta f_1 dt - \int \Delta f_2 dt) + 4.699 \Delta f_1 + 2.22 \Delta P_{g1} + 0.279 \Delta X_{gv1} - 4.71 \Delta f_2 - 2.24 \Delta P_{g2} - 0.284 \Delta X_{gv2} \quad .. \quad (6.18)$$

With a step load disturbance of 0.01 p.u. of power in Area 1, the responses of the state variables representing Δf_1 and Δf_2 are determined and given in Figure 6.2.

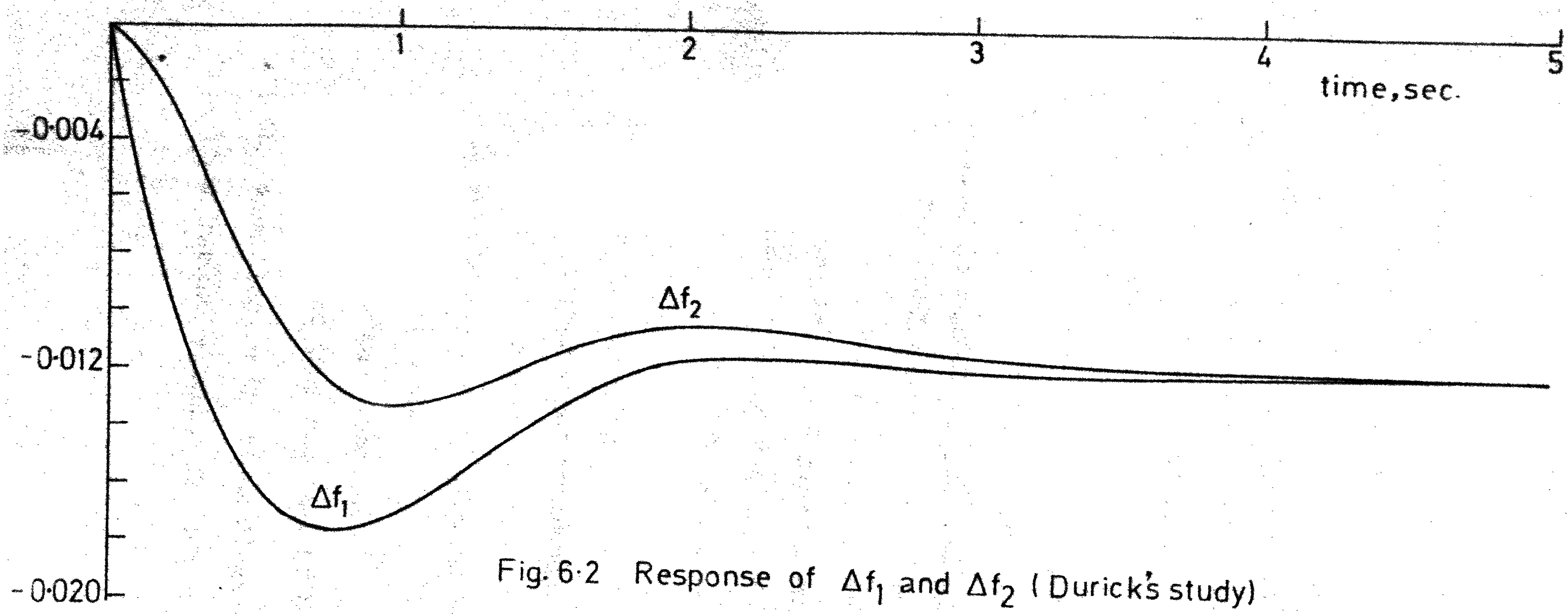


Fig. 6·2 Response of Δf_1 and Δf_2 (Durick's study)

6.3 TWO-AREA LFC SYSTEM WITH EXCITATION CONTROL

The block diagram of the two-area LFC system with excitation control included in one of the areas is given in Figure 6.3. The voltage input for the second area, however, remains the same as in Figure 6.1. Thus the effect of excitation control in one of the areas is studied in this section.

The salient differences between the block diagrams given in Figure 6.1 and Figure 6.3 are:

(a) Excitation control loop included in Area 1

The block diagram for the perturbation model of excitation loop has been derived from the standard IEEE block diagram for type 1 excitation system[30,31]. The same is shown in Figure 6.4 in its complete shape. The description of the variables and parameters in the same is

ΔV_T = terminal voltage

T_R = regulator input transducer time constant

K_A, T_A = amplifier gain and time constant

K_E = constant corresponding to setting of the shunt field rheostat

T_E = exciter time constant

K_F, T_F = gain and time constant of the stabilizer.

All the variables in the block diagram are perturbations from their nominal values. In this study the following

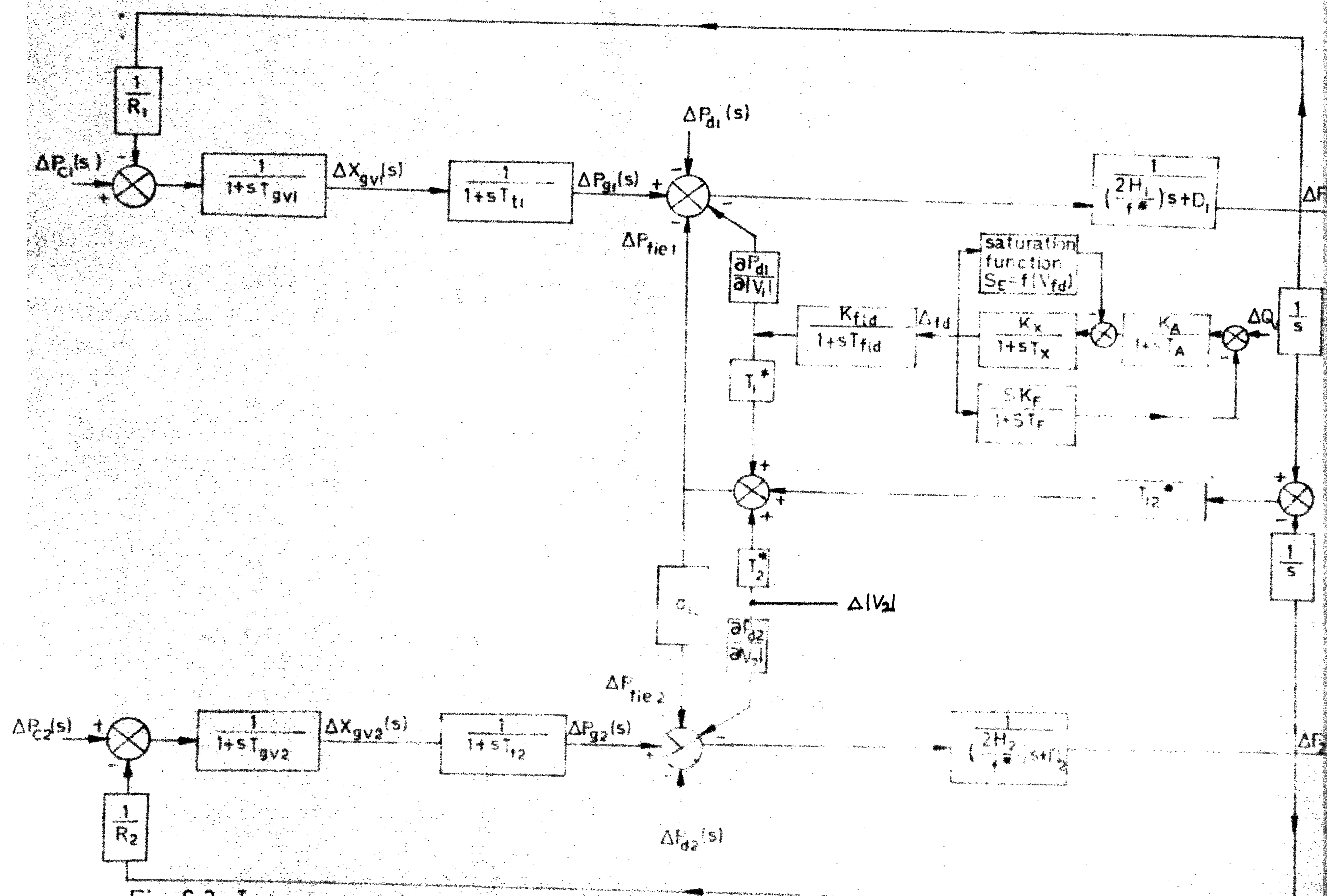


Fig. 6·3 Two-area system with excitation control in one area

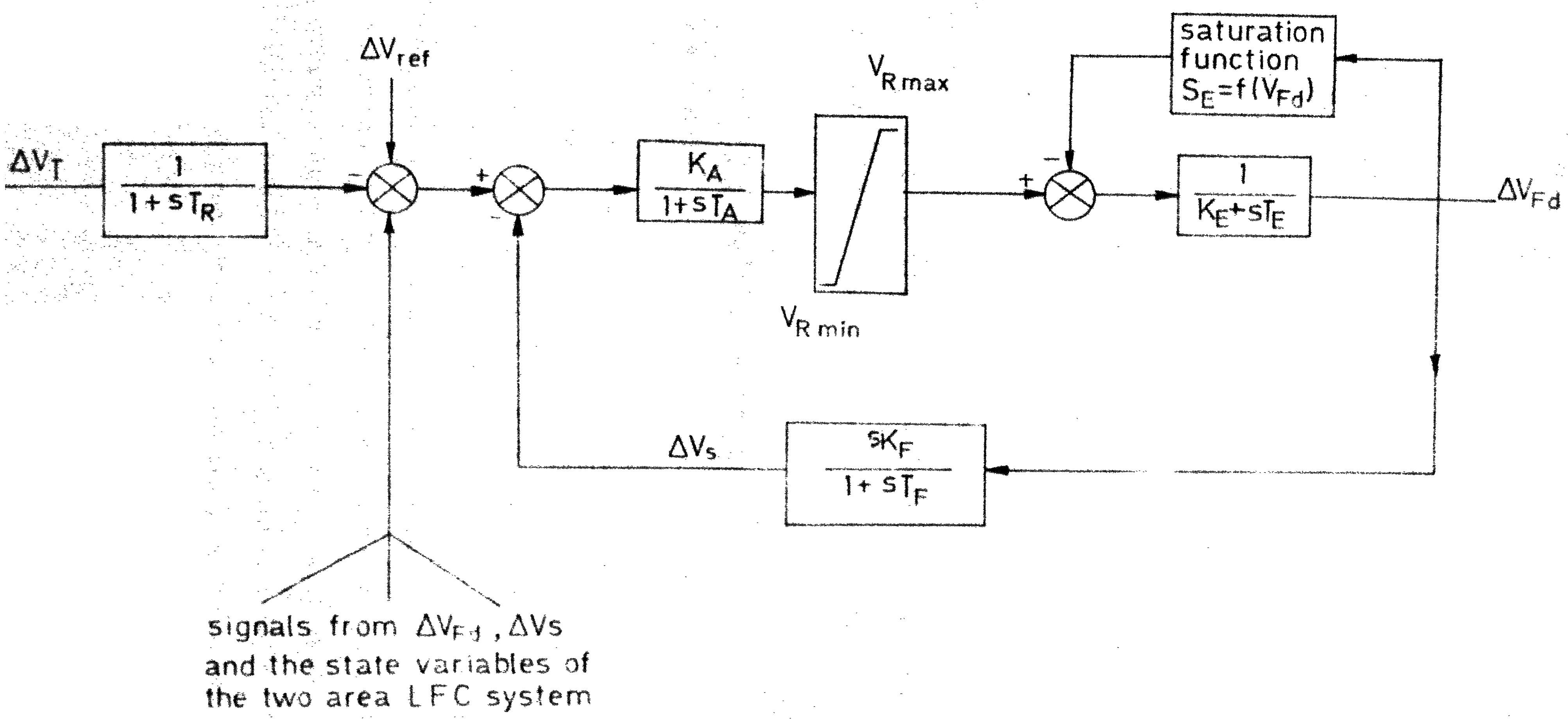


Fig.6·4 Perturbation model of an excitation control system

simplifying assumptions are made:

(i) $T_R = 0$

(ii) $T_A = 0$

(iii) the limiter after the amplifier block is considered to be absent.

(iv) the saturation function S_E is taken to be a constant multiplier. However, the ramifications of treating the saturation function S_E in its true shape are dealt with in Section 6.5.

Also, to fit the above model in that given in Figure 6.3 the following equalities are made:

$$K_X \text{ of Figure 6.3} = \frac{1}{K_E}\ , \quad T_X \text{ of Figure 6.3} = \frac{T_E}{K_E}\ ,$$
$$\Delta\left|V_1\right| \text{ of Figure 6.3} = \Delta V_T$$

where K_E, T_E and ΔV_T are described above.

(b) Interaction of voltage on load demand

In practical power systems the effect of voltage on load forms a major interaction. Hence this is taken into account by means of the constants $\partial P_{d1}/\partial\left|V_1\right|$ and $\partial P_{d2}/\partial\left|V_2\right|$ which represent the change of load per unit change in voltage in the Areas 1 and 2 respectively. A typical value of 1.0 is taken for each of these constants from the results obtained by Bogucki and Wojcik[35] who have conducted exhaustive tests on the Polish network for various kinds of loads.

6.4 APPLICATION OF OPTIMAL CONTROL THEORY

The dynamics of the system shown in Figure 6.3 can be given as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \Gamma\,\Delta\,\underline{P}_d \tag{6.19}$$

which, after defining the states with respect to their steady state values, can be written as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{6.20}$$

The state vector $\underline{x}$ in (6.20) consists of the 11-elements $\int \Delta f_1 dt$, Δf_1, ΔP_{g1}, ΔX_{gv1}, $\int \Delta f_2 dt$, Δf_2, ΔP_{g2}, ΔX_{gv2}, $\Delta |V_1|$, ΔV_{fd} and ΔV_s. The last three variables are perturbed value of terminal voltage of Area 1, exciter output voltage and stabilizer output respectively. The control vector $\underline{u}$ consists of the four-elements ΔP_{c1}, ΔP_{c2}, ΔQ_v and $\Delta |V_2|$. Also

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{gv2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_X}{T_X} & \frac{K_f K_X}{T_f T_X} \\ 0 & \frac{-f^* T_2^*}{2H_1} & 0 & 0 & 0 & \frac{-(a_{12} T_2^* + \frac{\partial P_{d2}}{\partial |V_2|}) f^*}{2H_1} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{6.21}$$

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-f^*T^*_{12}}{2H_1} & \frac{-f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 & \frac{f^*T^*_{12}}{2H_1} & 0 & 0 & 0 & \frac{-f^*\left(T^*_1 + \frac{\partial P_{d1}}{\partial |V_1|}\right)}{2H_1} & 0 & 0 \\
0 & 0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{T_{gv1}R_1} & 0 & \frac{-1}{T_{gv1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\frac{-a_{12}f^*T^*_{12}}{2H_2} & 0 & 0 & 0 & \frac{a_{12}f^*T^*_{12}}{2H_2} & \frac{-f^*D_2}{2H_2} & \frac{f^*}{2H_2} & 0 & \frac{-a_{12}T^*_{12}f^*}{2H_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{gv2}R_2} & 0 & \frac{-1}{T_{gv2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{fld}} & \frac{K_{fld}}{T_{fld}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-(1+K_XS_E)}{T_X} & \frac{-K_X}{T_X} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_f(1+K_XS_E)}{T_fT_X} & -\left(\frac{1}{T_f} + \frac{K_fK_X}{T_fT_X}\right)
\end{bmatrix}
$$

.... (6.22)

The numerical values for the exciter control parameters given in the A and B matrices are:

$S_E = 1.25$

$T_1^* = 0.05$ where $T_1^* = \dfrac{\partial P_{tie1}}{\partial |V_1|}$

$T_2^* = 0.05$ where $T_2^* = \dfrac{\partial P_{tie1}}{\partial |V_2|}$

$\dfrac{\partial P_{d1}}{\partial |V_1|} = \dfrac{\partial P_{d2}}{\partial |V_2|} = 1.0$ or 0.0 as the case may be.

Field time constant $T_{fld} = 6.17$

Field gain $K_{fld} = 0.57$

Exciter time constant $T_X = \dfrac{T_E}{K_E} = \dfrac{0.5}{-0.05} = -10.0$

Exciter gain $K_X = \dfrac{1}{K_E} = \dfrac{1}{-0.05} = -20.0$

Stabilizer time constant $T_F = 1.0$

Stabilizer gain $K_F = 0.04$

Amplifier gain $K_A^{\ddagger} = 25.0$; Amplifier time Constant $T_A = 0.0$

The rest of the parameters are as described in Appendix A.

The performance index for minimization is taken as

$$J = \int_0^\infty (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u})\, dt \tag{6.23}$$

Here Q is a 12x12 matrix with the elements Q(2,2) and Q (6,6) each equal to 1, the rest of the elements being being zero. This means that weightages are given to the state variables Δf_1, and Δf_2 only. Also in this study, four different values are used for R as detailed in Section 6.6.

---

‡ For convenience in computation this gain is incorporated into the exciter gain K_X making $K_X = -500.0$. This is done by shifting the line coming from S_E to the summing point ahead of the amplifier block.

Pontryagin's minimum principle is applied to find the closed loop controller is of the form

$$\underline{u}(t) = -K\,\underline{x}(t) = -R^{-1}\,B^T\,P\,\underline{x}(t) \tag{6.24}$$

where P (11x11) is the solution of the matrix Riccati equation

$$\dot{P} = -PA - A^TP + P\,B\,R^{-1}\,B^T\,P - Q \tag{6.25}$$

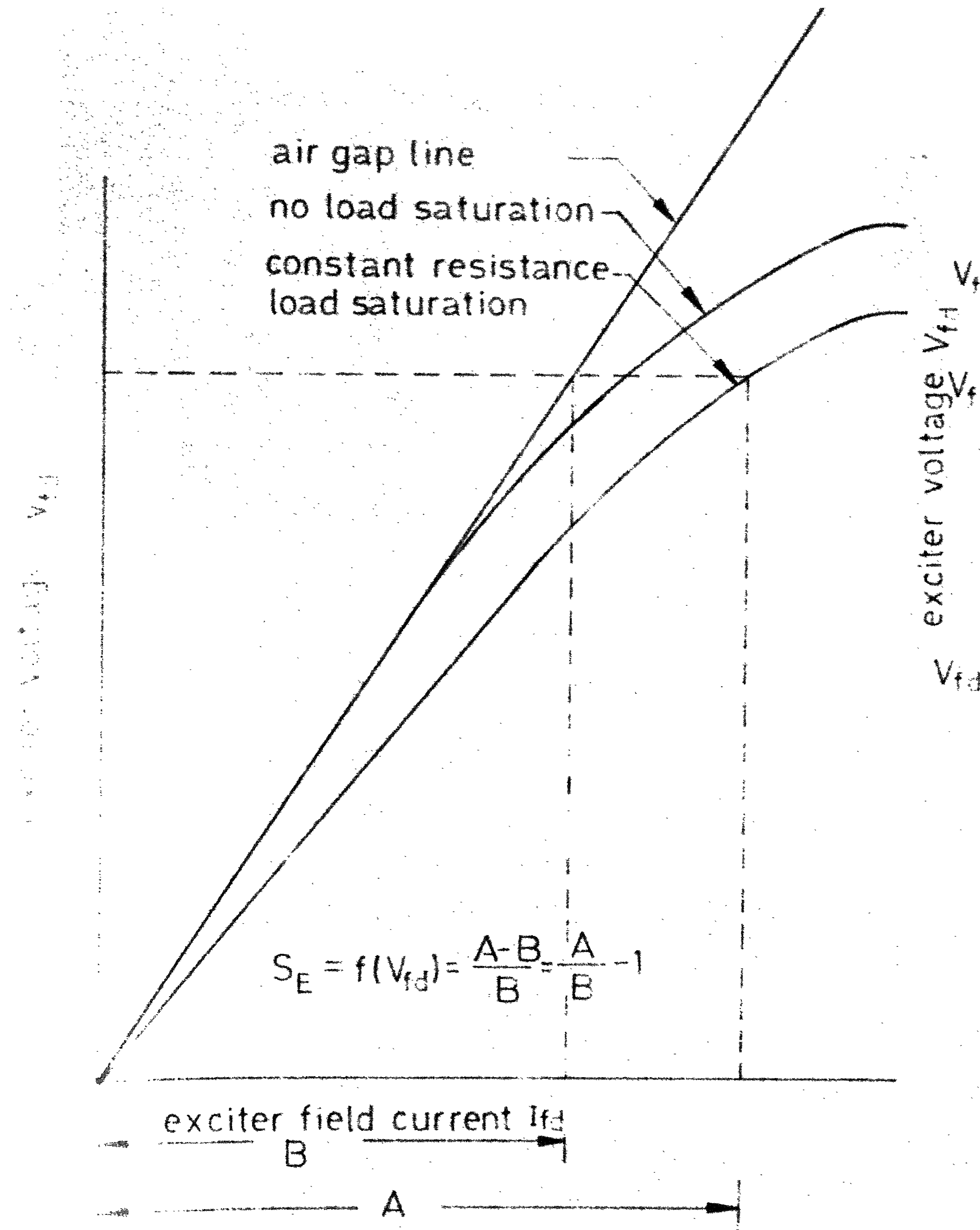
This matrix Riccati equation is solved by the Blackburn-Man[11,12] method given in Appendix C; however, the procedure given therein for determination of the initial solution takes considerable time on the computer for systems of order say 6 and above; hence the same cannot be used for the 11th order system studied here. Instead, the method of Kleinman[37] is used for finding the initial solution of the matrix Riccati equation. This method in turn requires the computation of e^{At} for which Vidyasagar's[36] algorithm is used. Both these algorithms are described in Appendix D. This procedure for determining the initial solution requires much less computer time as compared to that given at the end of Appendix C.

6.5 SATURATION NONLINEARITY

Referring to Figure 6.3, the block representing the saturation function S_E is a nonlinear function of V_{fd}. As we are dealing with linear analysis, inclusion of the saturation nonlinearity in its actual form is not possible. However, as it is only a mild type of nonlinearity, it can handled by the method of Gain Scheduling described below.

Gain Scheduling:

The method for calculating the saturation function S_E at a particular value of V_{fd} is shown in Figure 6.5A. Also let the saturation curve shown in Fig.6.5B be divided into, say, three equivalent linear portions OP, PQ and QR. It is easy to see that for each of the linear portions the value of S_E remains constant. Assuming a nominal value for V_{fd}, as long as the value for $(V_{fd} + \Delta\ V_{fd})$ is in a particular linear range (Figure 6.5B), the corresponding value of S_E is used for computing the elements of the matrix A; and the matrix Riccati equation (6.25) is solved. Thus the method reduces to that of solving three different matrix Riccati equations for the three values of S_E and these three solutions, say P_1, P_2, P_3 are used to determine the corresponding feedback gains K_1, K_2 and K_3. While determining the response of the system provision is made in the program to check at every step, in which of the linear portions the value of $(V_{fd} + \Delta\ V_{fd})$ is, and then the appropriate gain is used to compute the closed loop matrix $(A - B\ K_i)$; $i = 1,2,3$.



...5-A Exciter saturation curves showing ...ure for calculating the saturation function S_E

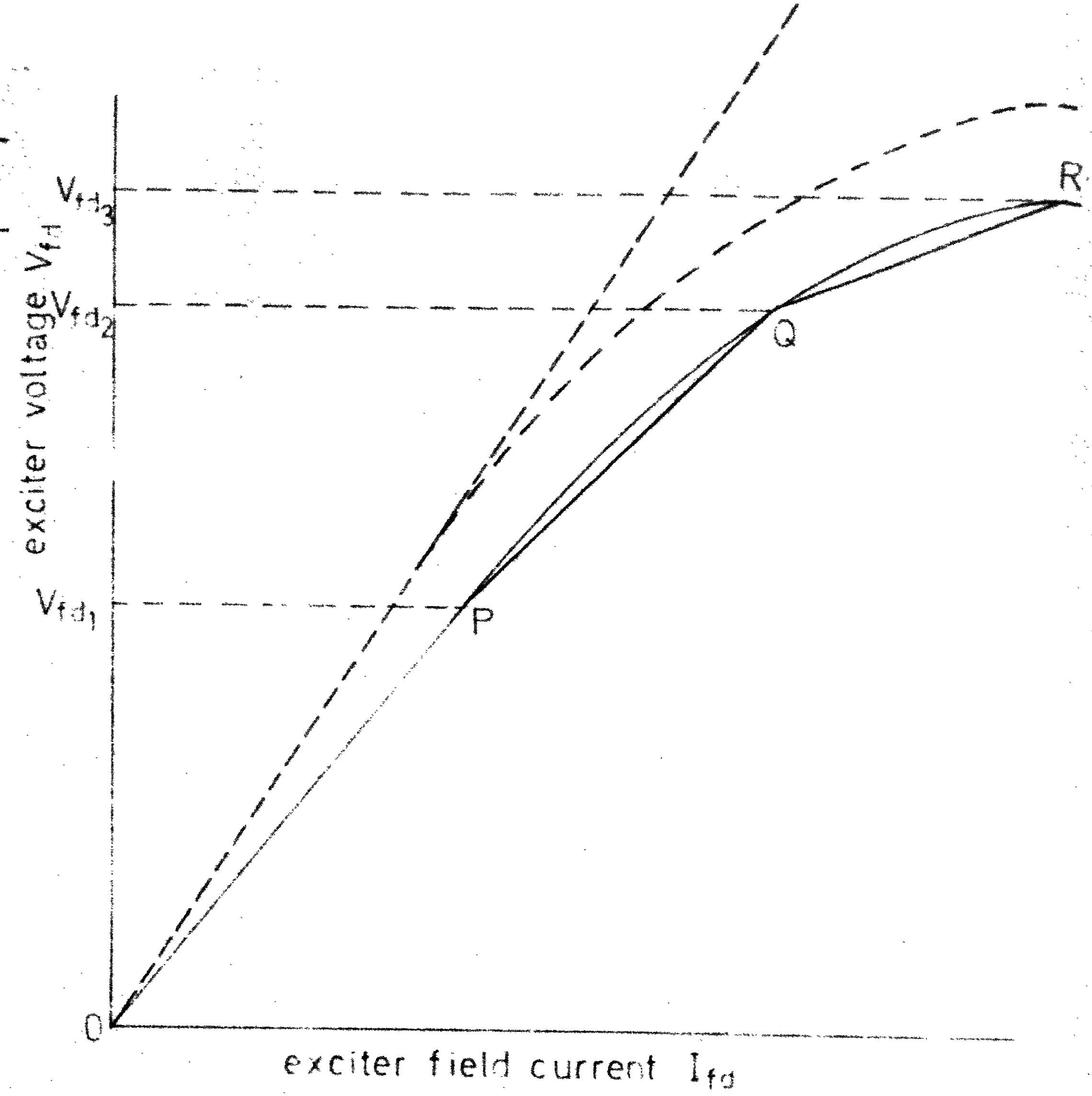


Fig. 6·5-B Equivalent exciter saturation curve used in the method of gain scheduling

6.6 COMPUTATIONAL RESULTS

Here the A matrix of (6.22) is computed as:

$$
\begin{bmatrix}
0.0 & 1.0000 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-3.2700 & -0.0500 & 6.0000 & 0.0 & 3.2700 & 0.0 & 0.0 & 0.0 & -6.3000 & 0.0 & 0.0 \\
0.0 & 0.0 & -3.3333 & 3.3333 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -5.2083 & 0.0 & -12.5000 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
3.2700 & 0.0 & 0.0 & 0.0 & -3.2700 & -0.0500 & 6.0000 & 0.0 & 0.3000 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.3333 & 3.3333 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -5.2083 & 0.0 & -12.5000 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1620 & 0.0924 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -2.4000 & -50.0000 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.0960 & -3.0000
\end{bmatrix}
$$

The B matrix of (6.21) is computed as

$$
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 12.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 50.0 & 2.0 \\
0.0 & -0.3 & 0.0 & 0.0 & 0.0 & -5.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}^T
$$

Four types of study are made in this chapter by employing four different combinations for the values of the parameter $\partial P_d / |\partial V_1|$ $(= \partial P_d / \partial |V_2|)$ and the weighting matrix R, keeping the rest of the parameters constant in all the studies.

a) $$\frac{\partial P_d}{\partial |V_1|} = \frac{\partial P_d}{\partial |V_2|} = 1.0, \quad R = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1 \end{bmatrix}$$

b) $$\frac{\partial P_d}{\partial |V_1|} = \frac{\partial P_d}{\partial |V_2|} = 0.0, \quad R = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1 \end{bmatrix}$$

c) $$\frac{\partial P_d}{\partial |V_1|} = \frac{\partial P_d}{\partial |V_2|} = 1.0, \quad R = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.01 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.01 \end{bmatrix}$$

d) $$\frac{\partial P_d}{\partial |V_1|} = \frac{\partial P_d}{\partial |V_2|} = 0.0, \quad R = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.01 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.01 \end{bmatrix}$$

The solution matrices (symmetric) P of the matrix Riccati equation (6.25) in the four cases respectively are

$$\begin{bmatrix}
0.2868 & -0.1207 & -0.1962 & -0.0468 & -0.2868 & 0.0010 & 0.0212 & 0.0058 & 0.3659 & 0.0051 & -0.0217 \\
 & 0.2936 & 0.2222 & 0.0399 & 0.1207 & -0.0084 & -0.0084 & -0.0013 & -0.2806 & -0.0010 & -0.0084 \\
 & & 0.2709 & 0.0589 & 0.1962 & -0.0029 & -0.0110 & -0.0025 & -0.4107 & -0.0033 & -0.0020 \\
 & & & 0.0137 & 0.0469 & -0.0 & -0.0024 & -0.0006 & -0.0959 & 0.0009 & 0.0003 \\
 & & & & 0.2868 & -0.0010 & -0.0212 & -0.0058 & -0.3659 & -0.0051 & 0.0217 \\
 & & & & & 0.0553 & 0.0142 & 0.0012 & -0.0016 & -0.0002 & 0.0016 \\
 & & & & & & 0.0159 & 0.0034 & 0.0166 & 0.0001 & 0.0007 \\
 & & & & & & & 0.0009 & 0.0042 & 0.0 & 0.0001 \\
 & & & & & & & & 0.7492 & 0.0102 & -0.0462 \\
 & & & & & & & & & 0.0003 & -0.0033 \\
 & & & & & & & & & & 0.0458
\end{bmatrix}$$

$$
\begin{bmatrix}
0.5199 & 0.0497 & -0.1658 & -0.0448 & -0.5199 & 0.0497 & 0.1658 & 0.0448 & 0.0434 & 0.0012 & -0.0126 \\
 & 0.3217 & 0.2459 & 0.0443 & 0.0497 & 0.0046 & 0.0249 & 0.0078 & -0.0061 & 0.0004 & -0.0057 \\
 & & 0.3168 & 0.0710 & 0.1658 & 0.0249 & 0.0281 & 0.0070 & -0.0164 & 0.0002 & -0.0049 \\
 & & & 0.0170 & 0.0448 & 0.0078 & 0.0070 & 0.0016 & -0.0043 & 0.0 & -0.0009 \\
 & & & & 0.5199 & -0.0497 & -0.1658 & -0.0448 & -0.0434 & -0.0012 & 0.0126 \\
 & & & & & 0.3217 & 0.2459 & 0.0443 & 0.0061 & -0.0004 & 0.0057 \\
 & & & & & & 0.3168 & 0.0710 & 0.0164 & -0.0002 & 0.0049 \\
 & & & & & & & 0.0170 & 0.0043 & -0.0 & 0.0009 \\
 & & & & & & & & 0.0039 & 0.0001 & -0.0008 \\
 & & & & & & & & & 0.0 & -0.0001 \\
 & & & & & & & & & & 0.0019
\end{bmatrix}
$$

$$
\begin{bmatrix}
0.1352 & -0.1078 & -0.1219 & -0.0254 & -0.1352 & 0.0044 & 0.0078 & 0.0017 & 0.1867 & 0.0017 & -0.0040 \\
 & 0.2491 & 0.1658 & 0.0276 & 0.1078 & -0.0106 & -0.0083 & -0.0014 & -0.2149 & -0.0011 & -0.0008 \\
 & & 0.1653 & 0.0326 & 0.1219 & -0.0068 & -0.0084 & -0.0017 & -0.2348 & -0.0017 & 0.0005 \\
 & & & 0.0069 & 0.0254 & -0.0011 & -0.0017 & -0.0003 & -0.0485 & -0.0004 & 0.0002 \\
 & & & & 0.1352 & -0.0044 & -0.0078 & -0.0017 & -0.1867 & -0.0017 & 0.0040 \\
 & & & & & 0.0180 & 0.0020 & 0.0001 & 0.0084 & 0.0 & 0.0001 \\
 & & & & & & 0.0021 & 0.0004 & 0.0120 & 0.0001 & 0.0 \\
 & & & & & & & 0.0001 & 0.0025 & 0.0 & 0.0 \\
 & & & & & & & & 0.3563 & 0.0032 & -0.0067 \\
 & & & & & & & & & 0.0 & -0.0004 \\
 & & & & & & & & & & 0.0049
\end{bmatrix}
$$

$$
\begin{bmatrix}
0.3639 & -0.0236 & -0.0947 & -0.0260 & -0.3639 & 0.0236 & 0.0947 & 0.0260 & 0.0295 & 0.0010 & -0.0109 \\
 & 0.2606 & 0.1894 & 0.0338 & 0.0236 & 0.0657 & 0.0814 & 0.0183 & -0.0030 & 0.0002 & -0.0026 \\
 & & 0.2425 & 0.0545 & 0.0947 & 0.0814 & 0.1024 & 0.0235 & -0.0091 & 0.00 & -0.0016 \\
 & & & 0.0132 & 0.0260 & 0.0183 & 0.0235 & 0.0054 & -0.0025 & 0.0 & -0.0003 \\
 & & & & 0.3639 & -0.0236 & -0.0947 & -0.0260 & -0.0295 & -0.0010 & 0.0109 \\
 & & & & & 0.2606 & 0.1894 & 0.0338 & 0.0030 & -0.0002 & 0.0026 \\
 & & & & & & 0.2425 & 0.0545 & 0.0091 & 0.0 & 0.0016 \\
 & & & & & & & 0.0132 & 0.0025 & 0.0 & 0.0003 \\
 & & & & & & & & 0.0026 & 0.0001 & -0.0007 \\
 & & & & & & & & & 0.0 & -0.0001 \\
 & & & & & & & & & & 0.0013
\end{bmatrix}
$$

In each of the four cases above, the closed loop system matrix $(A-B R^{-1} B^T P)$ is computed. The responses of the frequency deviations Δf_1 and Δf_2 obtained for the cases (a) and (b) are given in Figure 6.6 as continuous and dashed curves respectively. Also the responses obtained for the cases (c) and (d) are given in Figure 6.7 as continuous and dashed curves respectively.

6.7 CONCLUSIONS

The effect of excitation control in one area of a two-area LFC system is studied. Figures 6.6 and 6.7 summarize the results obtained. The system studied by Durick and responses obtained therein are given in Figure 6.2 for a comparative study.

The following conclusions are drawn from the responses obtained. The interaction of voltage on load demand prevents the system from reaching at least a non-zero steady state and thus has a deteriorating effect on the system response. Decreasing the penalty factors on the two inputs corresponding to exciter control in Area 1 and voltage perturbation in Area 2 will prove to be much harmful to the system.

In the study made with the assumption that there is no interaction of voltage on load demand, the steady state frequency error is lesser than that obtained in Durick's study (Figure 6.2) This is explainable to be

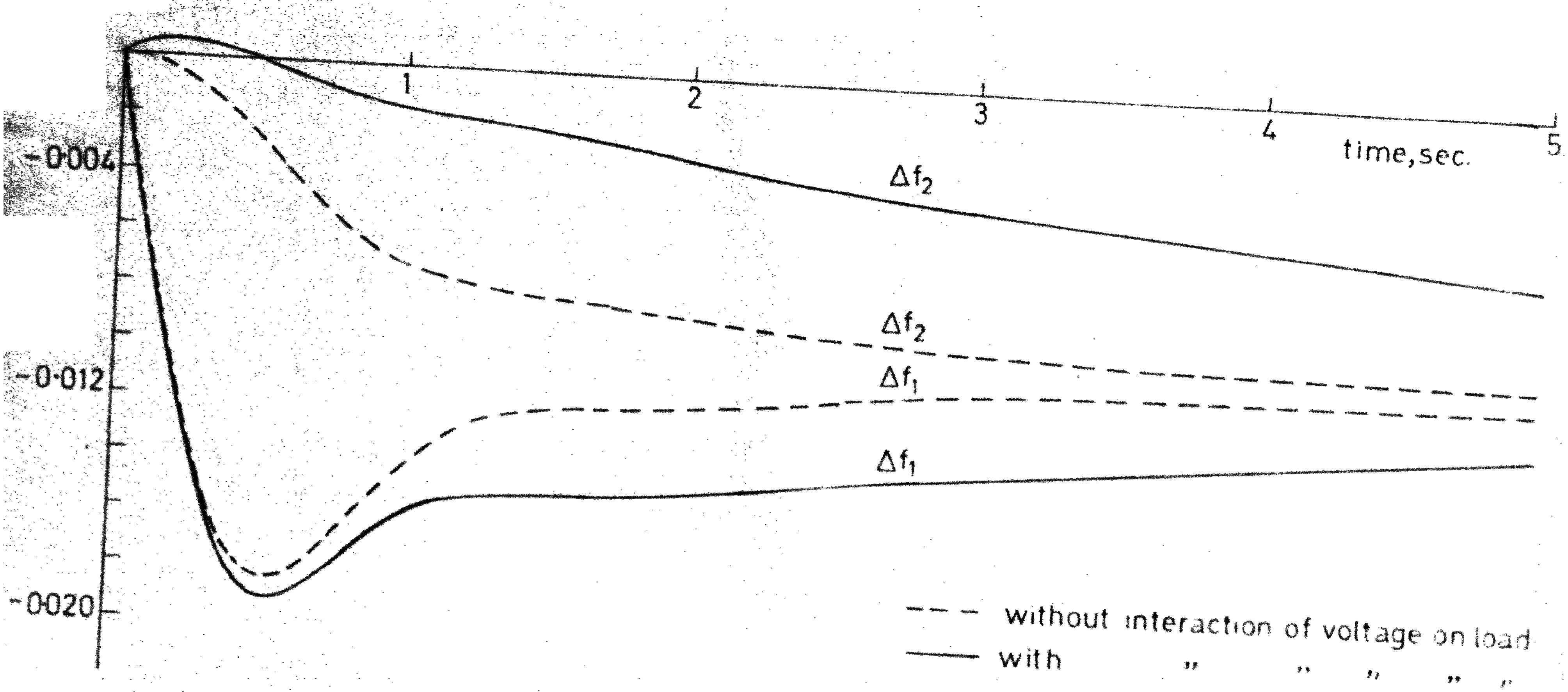


Fig. 6.6 Response of Δf_1 and Δf_2 with R = diag(1,1,0·1,0·1)

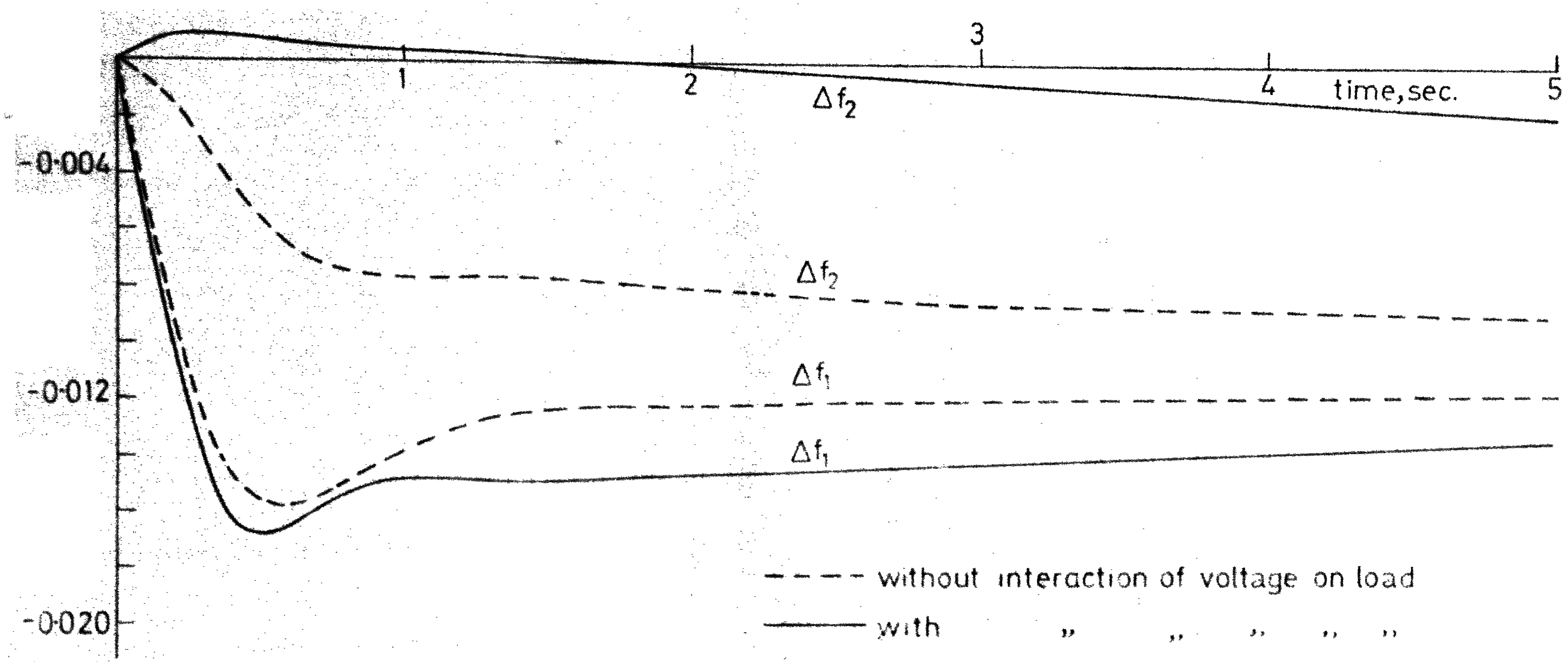


Fig. 6·7 Response of Δf_1 and Δf_2 with R = diag(1,1,0·01, 0·01)

due to inclusion in this study, of the usual load frequency controllers ΔP_{c1} and ΔP_{c2}.

A comparison of the responses given in Figures 6.6 and 6.7 with those obtained by Elgerd and Fosha[3] emphasize the optimistic nature of the results obtained with the assumption of noninteraction between Megawatt frequency and Megawar voltage control loops. The above results also confirm that in LFC systems where the inherent stability of the system (without considering exciter loop) is only marginal, inclusion of exciter loop will actually lead to instability of the system. Thus the study made in this chapter, is of great importance because it indicates the necessity or otherwise, of employing compensating networks that will stabilize the system.

CHAPTER 7

NONMEASURABILITY OF STATES AND CONSTRUCTION OF AN OBSERVER

The optimal closed loop controller designed by Elgerd and Fosha[3] for a two area system is based on the assumption that all the state variables are available for feedback. In a practical system measurement of some of the state variables may be either extremely difficult or altogether impossible because of limitations in the measuring apparatus. One method of overcoming such a difficulty is, constructing an observer of the Luenberger type[18,19]. By this means, an observer is constructed for the inaccessible state variables which are then used in the optimum regulator configuration for finding the responses of the state variables. Bongiorno and Youla[8] have given the theory of construction of a compatible observer of the Luenberger type. Section 7.2 gives a summary of results obtained by them. Section 7.3 deals with the computational results for the construction of an observer for the two-area LFC system.

7.2 THEORY OF CONSTRUCTION OF A COMPATIBLE OBSERVER (LUENBERGER TYPE)

Let the system dynamic equation and output equation be given by

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{7.1}$$

and

$$\underline{y} = D\,\underline{x} \tag{7.2}$$

Here $\underline{x}$ is an n-vector

$\underline{y}$ is a p-vector

$\underline{u}$ is an r-vector.

A is an (nxn) system matrix

B is an (nxr) input distribution matrix

D is a (pxn) measurement matrix.

D is constructed to have a simple structure; each row consists of 1 in only one place corresponding to the observable state and the rest of the elements in the row are zero.

For this system, the dynamic observer is defined as

$$\dot{\underline{v}} = H\,\underline{v} + S\,\underline{y} + E\,\underline{u} \tag{7.3}$$

where $\underline{v}$ is an (n-p)-vector

H is an (n-p)x(n-p)-matrix

S is an (n-p) x p matrix

E is an (n-p) x r matrix.

H is chosen to be a stable matrix with eigenvalues different from those of the system matrix A of (7.1).

Using this observer, the estimate $\hat{\underline{x}}$ of the state vector $\underline{x}$ is given as

$$\hat{\underline{x}} = L_1\,\underline{y} + L_2\,\underline{v} \tag{7.4}$$

where L_1 is an nxp matrix

L_2 is an nx(n-p) matrix.

Definition:

An observer is said to be compatible if its output equals the state of the plant, to within an exponentially decaying error.

For the observer to be compatible, it is necessary that (a) the plant be completely observable and (b) the observer be completely controllable with respect to the plant output. Mathematically, condition (a) means that the matrix

$$\Delta_o = \left[D^T \vdots A^T D^T \vdots (A^T)^2 D^T \vdots \ldots \vdots (A^T)^{n-1} D^T \right] \quad (7.5)$$

should be of rank n; and condition (b) means that the matrix

$$\Gamma_c = \left[S \vdots H S \vdots H^2 S \vdots \ldots \vdots H^{n-1} S \right] \quad (7.6)$$

should be of rank (n-p).

The observation vector $\underline{v}$ is related to the state vector $\underline{x}$ by

$$\underline{v} = U \underline{x} + \underline{e} \quad (7.7)$$

where U is the unique solution of

$$U A - H U = S D \quad (7.8)$$

provided A and H do not have common eigenvalues. For a choice of E = U B, the vector $\underline{e}$ in (7.7) is an exponentially decaying error between the actual and observed state vectors.

The differential equation for this error is given by

$$\dot{\underline{e}}(t) = H\,\underline{e}(t) \tag{7.9}$$

The solution of the equation (7.9) is

$$\underline{e}(t) = \exp\,(H(t-t_o))\underline{e}(t_o) \tag{7.10}$$

The reconstructed state vector is now given by

$$\hat{\underline{x}} = L(W\,\underline{x} + \hat{\underline{e}}) \tag{7.11}$$

where

$$\hat{\underline{e}} = \begin{bmatrix} \underline{0}_p \\ \hline \underline{e} \end{bmatrix} = \begin{bmatrix} \underline{0}_p \\ \hline \exp(H(t-t_o))\underline{e}(t_o) \end{bmatrix} \tag{7.12}$$

$\underline{0}_p$ in the above equation denotes a p-element zero vector. The matrices L and W can be split up as

$$L = \left[L_1 \;\vdots\; L_2\right] \tag{7.13}$$

and

$$W = \begin{bmatrix} D \\ \hline U \end{bmatrix} \tag{7.14}$$

$$U = -\phi_o^{-1}(H)\,\hat{U} \tag{7.15}$$

and

$$\hat{U} = \Gamma_c \quad \Omega\,\Delta_o \tag{7.16}$$

where Δ_o and Γ_c are defined in (7.5) and (7.6) respectively. Also

$$\phi_o(\lambda) = \sum_{i=0}^{n} \alpha_i \quad \lambda_i \qquad \lambda^i \tag{7.17}$$

with $\alpha_n = 1$, is the characteristic equation of the matrix A and α_i's are its coefficients. The matrix Ω in (7.16) is defined as

$$\Omega = \begin{bmatrix} \alpha_1 \underline{I}_p & \alpha_2 \underline{I}_p & \cdots & \alpha_{n-1} \underline{I}_p & \alpha_n \underline{I}_p \\ \alpha_2 \underline{I}_p & \alpha_3 \underline{I}_p & \cdots & \alpha_n \underline{I}_p & \underline{0}_{pp} \\ \vdots & & & & \vdots \\ \alpha_{n-1} \underline{I}_p & \alpha_n \underline{I}_p & \cdots & \underline{0}_{pp} & \underline{0}_{pp} \\ \alpha_n \underline{I}_p & \underline{0}_{pp} & \cdots & \underline{0}_{pp} & \underline{0}_{pp} \end{bmatrix} \tag{7.18}$$

Here $\underline{I}_p$ and $\underline{0}_{pp}$ denote pxp identity matrix and pxp null matrix respectively.

For the observer to be compatible, it is required that the matrix W should be nonsingular. With this condition, L is computed in the simplest manner as equal to W^{-1} and the estimate $\hat{\underline{x}}$ of the vector $\underline{x}$ computed as in (7.4).

7.3 APPLICATION TO A LOAD FREQUENCY CONTROL SYSTEM - COMPUTATIONAL RESULTS

Figure 7.1 gives the block diagram of a two-area LFC system. The description of the matrices A (9x9) and B (9x2) of (7.1) is as given in (A-8) and (A-9) of Appendix A; also they are computed as given in Section 3.7 of Chapter 3. The matrix D (4x9) of (7.2) is taken as

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

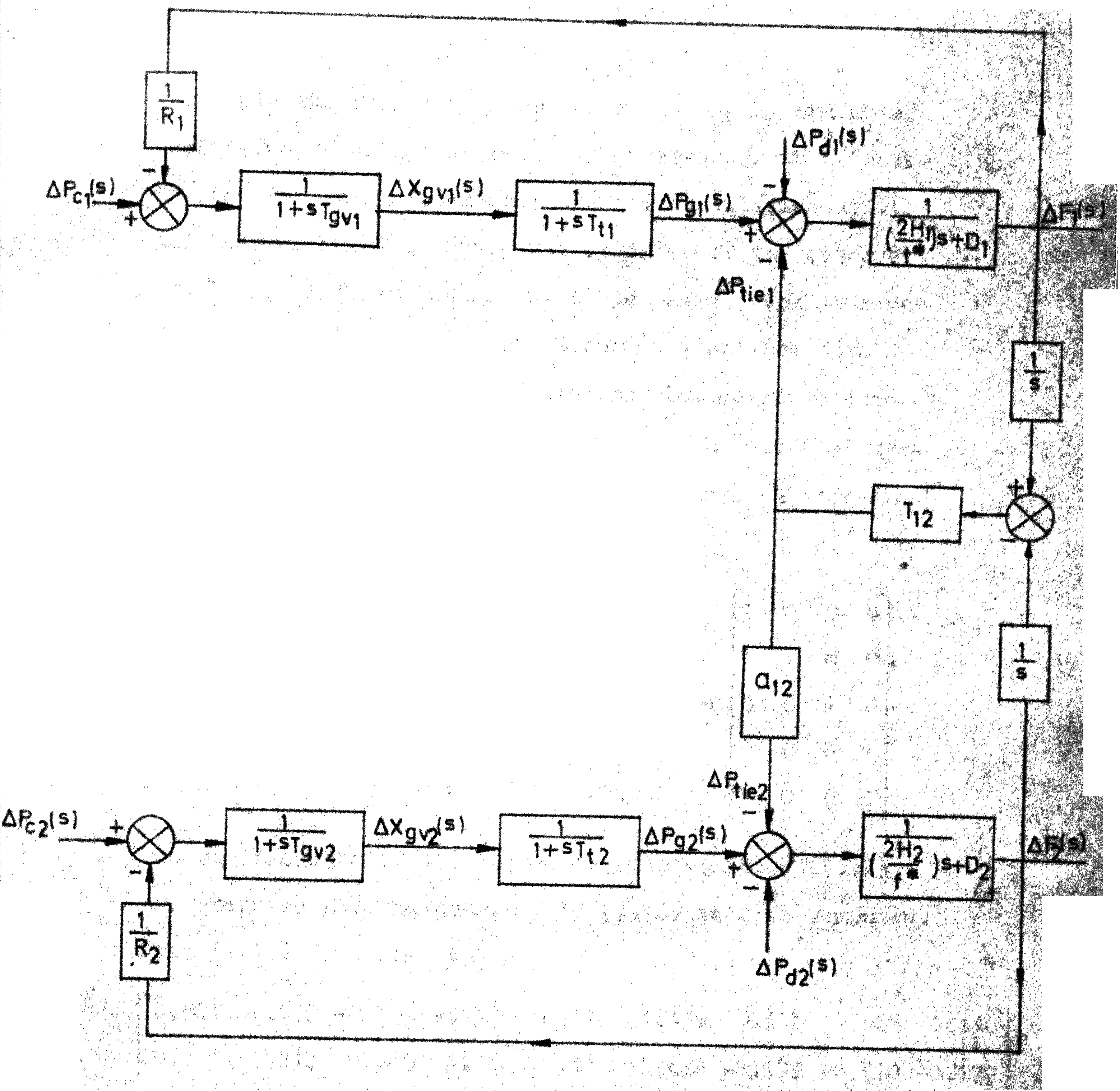


Fig. 7·1 Block diagram of a two area L.F.C.system

Thus only the four state variables x_1, x_2, x_4 and x_7 are considered to be measurable. An observer is to be constructed for this system from the four available outputs and the two inputs. Hence the dynamics of the observer should be of fifth order. As stated above, the matrices H and S of the observer are so chosen that they form a controllable pair(H,S) and also that the matrix H does not have any eigenvalue in common with the original system matrix A. Keeping these considerations in view the matrices H and S are chosen as

$$H = \begin{bmatrix} -1.72 & 0 & 0 & 0 & 0 \\ 0 & -1.87 & 0 & 0 & 0 \\ 0 & 0 & -1.99 & 0 & 0 \\ 0 & 0 & 0 & -2.15 & 0 \\ 0 & 0 & 0 & 0 & -2.33 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The coefficients of the characteristic equation (7.17) are computed and the matrix $\hat{U}$ of (7.16) is also computed. From (7.15) U is computed as

$$\begin{bmatrix} 0.581 & -1.539 & -0.356 & -1.325 & -0.410 & 1.539 & 0.356 & 1.325 & 0.410 \\ 0.0 & 1.213 & 0.249 & 1.020 & 0.320 & -0.678 & -0.139 & -0.570 & -0.179 \\ 0.0 & 0.939 & 0.399 & 1.037 & 0.329 & -0.939 & -0.173 & -0.771 & -0.245 \\ 0.0 & 0.452 & 0.071 & 0.358 & 0.115 & -0.452 & -0.227 & -1.150 & -0.370 \\ 0.429 & -0.155 & -0.020 & -0.117 & -0.038 & 0.155 & 0.020 & 0.117 & 0.038 \end{bmatrix}$$

The matrix E is computed from the relation $E = U\ B$ as

$$E = \begin{bmatrix} -5.121 & 5.121 \\ 3.997 & -2.234 \\ 4.111 & -3.058 \\ 1.441 & -4.628 \\ -0.480 & 0.480 \end{bmatrix}$$

The matrix W is also calculated as in (7.14); and finally L which is the inverse of W^{-1} is computed. For use in (7.4) L is split up as L_1 and L_2. Here

$$L_1 = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ -6.044 & 2.046 & 0.020 & -0.005 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.008 & -5.330 & -3.204 & -0.001 \\ -3.966 & -0.022 & -0.022 & -0.002 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 35.160 & 2.403 & 1.005 & -1.977 \\ -105.479 & -7.482 & -3.121 & 5.527 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.815 & -3.825 & 6.267 & -0.045 & 12.979 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.668 & 9.968 & -4.836 & -0.008 & -2.278 \\ 0.809 & 1.912 & -0.242 & 0.977 & 8.144 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 16.217 & -4.493 & 11.925 & 1.455 & -103.892 \\ -50.668 & 13.991 & -37.043 & -8.423 & 314.404 \end{bmatrix}$$

The dynamical equations of the original system (7.1) and the observer system (7.3) are then solved for obtaining the response with inputs u_1 and u_2 given in terms of $\hat{\underline{x}}$ of (7.4). An initial condition $x_4(o) = -0.01$ is taken for the original system and a corresponding one for the observer. The coefficients for the optimal controller structure are computed by applying the Pontryagin's minimum principle (Appendix B) to the system under consideration. As the system taken in this study is identical to that taken by Elgerd and Fosha[3], the feedback coefficients computed by them for the optimal controller structure, are utilized here.

In the two-area LFC system the state variables of interest are the frequency deviations in the two areas Δf_1 and Δf_2 and the tieline interchange error ΔP_{tie1}. The responses of these state variables are determined (Figure 7.2) by cascading the observer system with the original system. Figure 7.3 gives the response of the above state variables under the assumption of complete state feedback[3].

7.4 COMMENTS AND CONCLUSIONS

In this chapter an observer of the Luenberger type is designed for the two-area LFC system assuming that only four out of the nine state variables are measurable. The estimate of the state vector thus obtained is utilized for

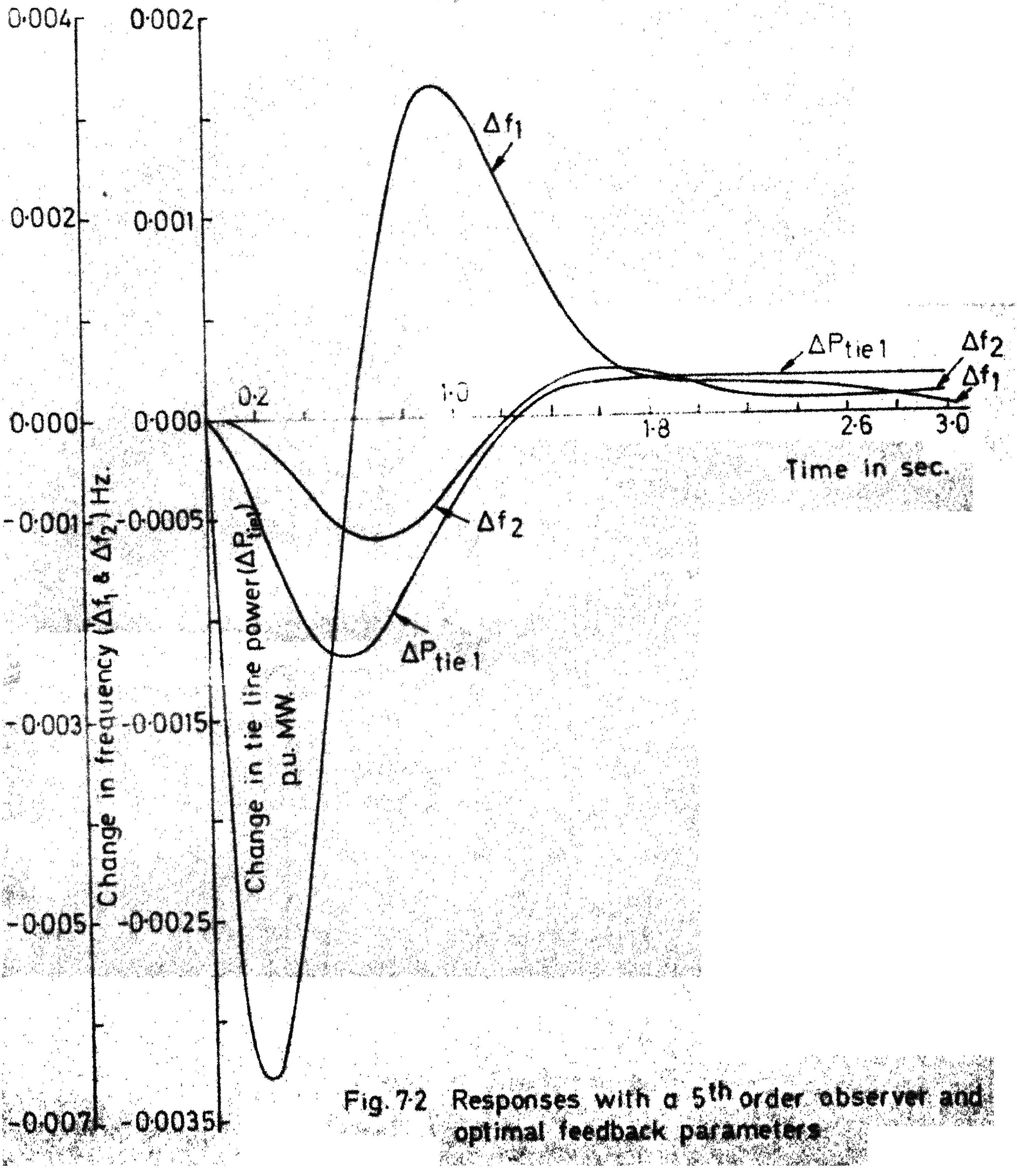


Fig. 7·2 Responses with a 5th order observer and optimal feedback parameters

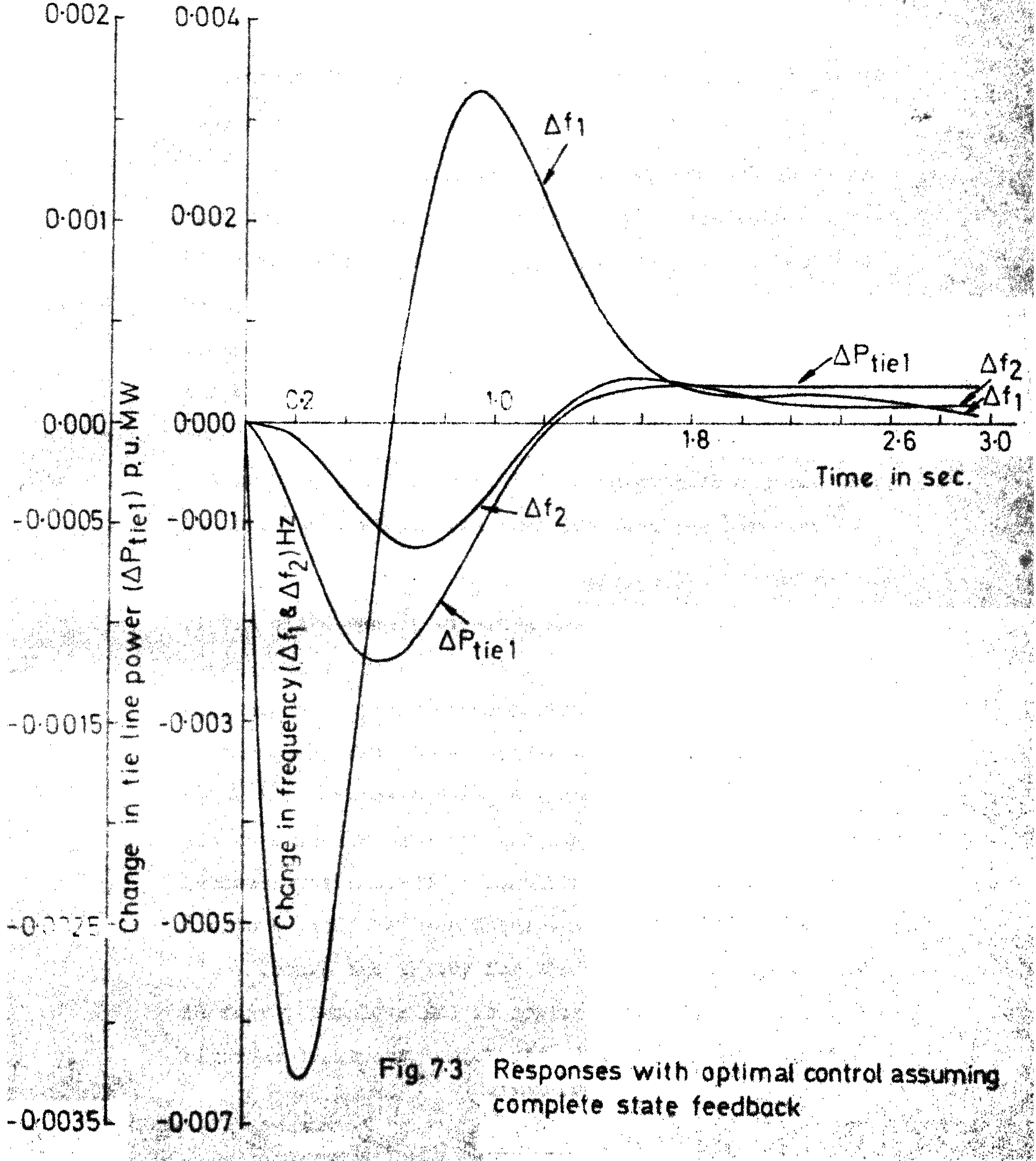


Fig. 7·3 Responses with optimal control assuming complete state feedback

determining the response of the system; the response with complete measurement is also given for comparison.

It is seen from the Figures 7.2 and 7.3 that the responses obtained by using an observer are almost identical to those obtained by assuming complete state feedback, and hence the method is advantageous in load frequency control of multi-area systems. However, when the observer system is of high order its cost will be proportionally high. Under such a situation, it is better to go in for some kind of suboptimal control using the available state variables for feedback purposes[41].

The theory of observers ensures the reconstruction of the state vector within a small error under certain conditions. It is desirable to keep the eigenvalues of the observer system dynamics, to be reasonably large so as to ensure fast decay of any possible error in the initial condition taken. A second requirement is that the observer should have smaller eigenvalues in order that its hardware realization is feasible. A suitable compromise has to be made between these two conflicting requirements.

Though the theory for the construction of an observer is fairly complete and is applicable to LFC systems, a more rigorous treatment, which also takes into account, the

physical realities, will be, to consider noise in the system dynamics as well as in measurement. For optimally controlling such linear stochastic LFC systems, well established stochastic control techniques are available[33] and have since been applied to small order LFC systems[34].

CHAPTER 8

CONCLUSIONS AND SUGGESTIONS

State variable approach and optimal control techniques are now extensively used for the analysis of LFC systems. However, when they are applied to higher order systems, computational effort and time increase considerably. The methods presented in this thesis, for the construction of suboptimal controllers, will enable one to overcome this difficulty and thus will enhance the usefulness of optimal control techniques in the area of Load Frequency Control.

The methods given for the model reduction of linear LFC systems are both elegant and efficient. In the case of suboptimal control by aggregation, it is also possible to correctly and quickly check the accuracy of representation by computing the degradation in performance.

Miniesy and Bohn[25] have constructed a suboptimal controller having a two level structure for the large signal model of a multiarea LFC system. For the first time, an attempt is made here for the reduction of the large signal model of a multiarea LFC system to a lower order nonlinear model and also for the construction of a suboptimal controller in the shape of the optimal controller, determined by Lukes' method, for the reduced model.

The studies made here by including excitation control loop in LFC systems not only reveal the behaviour of the system under realistic conditions but they also confirm the highly optimistic nature of the results obtained by the assumption of noninteraction of the Megawatt frequency and Megavar voltage control loops[3]. As such these studies are very helpful in taking appropriate decisions regarding provision of compensating networks for systems in which the open loop system matrix, without the inclusion of exciter loop, is marginally stable.

The results obtained with the construction of an observer for an LFC system are quite satisfactory. However, the cost of construction of an observer increases in proportion with the order of the observer system which in turn depends on the order of the multiarea system; as such the utility of the method is confined to small order multiarea systems only.

Some of the related problems for further research are:

(i) Optimal deterministic control of multiarea LFC systems with the inclusion of both the governor and tieline power nonlinearities.

(ii) Seeking reduction techniques for the optimal control of LFC systems, consisting of many areas (say 6 and above) which are different in their size (MW capacity) and/or parameters.

(iii) Extending the theory of aggregation due to Aoki for the reduction in computation of optimal controllers for linear stochastic multiarea LFC systems.

(iv) Suboptimal stochastic control of the large signal model of an LFC system having tieline power nonlinearity.

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APPENDIX A

TWO-AREA LOAD FREQUENCY CONTROL SYSTEM

The two-area load frequency control system considered by Elgerd and Fosha[3] is taken up for study. The block diagram for the two-area system is given in Figure 7.1. The system differential equations are:

$$\frac{d}{dt}\left(\int \Delta P_{tie1}\, dt\right) = T_{12}^{*}\left(\int \Delta f_1 dt - \int \Delta f_2 dt\right) \qquad (A.1)$$

$$\frac{d}{dt}\left(\int \Delta f_i dt\right) = \Delta f_i \qquad (A.2)$$

$$\frac{2H_i}{f^*} \frac{d}{dt} \Delta f_i + D_i \Delta f_i + \Delta P_{tie\,i} = \Delta P_{gi} - \Delta P_{di} \qquad (A.3)$$

$$\frac{d}{dt} \Delta P_{gi} = -\frac{1}{T_{ti}} \Delta P_{gi} + \frac{1}{T_{ti}} \Delta X_{gvi} \qquad (A.4)$$

$$\frac{d}{dt} \Delta X_{gvi} = -\frac{1}{T_{gvi}} \Delta X_{gvi} - \frac{1}{T_{gvi} R_i} \Delta f_i + \frac{1}{T_{gvi}} \Delta P_{ci}$$

$$\ldots\ i = 1,2 \qquad \ldots (A.5)$$

$$\Delta P_{tie2} = a_{12} \Delta P_{tie1} = -\frac{P_{r1}}{P_{r2}} \Delta P_{tie1}$$

In view of this relation, $\int \Delta P_{tie2}\, dt$ is not considered as a state variable.

$$T_{12}^{*} = \frac{|V_1|\,|V_2|}{X_{12} P_{r1}} \cos(\delta_1^{*} - \delta_2^{*})$$

$\delta_{12}^{*} = \delta_1^{*} - \delta_2^{*} = 30°$

$f^* = 60$ Hz

$H_1 = H_2 = 5$ sec.

$T_{gv1} = T_{gv2} = 0.08$ sec.

$R_1 = R_2 = 2.4$ Hz/p.u. MW

$T_{12}^{*} = 0.545$ p.u. MW

$P_{r1} = P_{r2} = 2000$ MW

$D_1 = D_2 = 8.33 \times 10^{-3}$ p.u.MW/Hz $\quad \Delta P_{d1} = 0.01$ p.u. MW

$T_{t1} = T_{t2} = 0.3$ sec. $\quad \Delta P_{d2} = 0.0$ p.u. MW

Equations (A.1) to (A.5) can be put in the form of vector differential equation as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \Gamma \Delta \underline{P}_d \tag{A.6}$$

where $\underline{x}$ is a 9 element vector consisting of the state variables $\int \Delta P_{tie1} dt$, $\int \Delta f_1 dt$, Δf_1, ΔP_{g1}, ΔX_{gv1}, $\int \Delta f_2 dt$, Δf_2, ΔP_{g2} and ΔX_{gv2}, and $\underline{u}$ is a 2-element vector consisting of the control variables ΔP_{c1} and ΔP_{c2}. Also $\Delta \underline{P}_d$ is a 2-element vector consisting of the two load disturbances ΔP_{d1} in Area 1 and ΔP_{d2} in Area 2 as its elements. Γ is a load disturbance distribution matrix given as

$$\Gamma = \begin{bmatrix} 0 & 0 & -\frac{f^*}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{f^*}{2H_2} & 0 & 0 \end{bmatrix}^T$$

By redefining the state and control vectors in terms of their steady state values, the terms $\Gamma \Delta \underline{P}_d$ in (A.6) can be eliminated and the equation (A.6) can be written as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{A.7}$$

Here A =

$$
\begin{bmatrix}
0 & T^*_{12} & 0 & 0 & 0 & -T^*_{12} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-f^*T^*_{12}}{2H_1} & \frac{-f^*D_1}{2H_1} & \frac{f^*}{2H_1} & 0 & \frac{f^*T^*_{12}}{2H_1} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-1}{T_{gv1}R_1} & 0 & \frac{-1}{T_{gv1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & \frac{-a_{12}f^*T^*_{12}}{2H_2} & 0 & 0 & 0 & \frac{a_{12}f^*T^*_{12}}{2H_2} & \frac{-f^*D_2}{2H_2} & \frac{f^*}{2H_2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{gv2}R_2} & 0 & \frac{-1}{T_{gv2}}
\end{bmatrix}
\quad \text{.. (A.8)}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & \frac{1}{T_{gv1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{gv2}}
\end{bmatrix}^T \qquad \text{(A.9)}
$$

For applying optimal control theory to the above system the performance to be minimized is taken as

$$
J = \int_0^\infty (\underline{x}^T Q \, \underline{x} + \underline{u}^T R \, \underline{u}) \, dt \qquad \text{(A.10)}
$$

APPENDIX B

OPTIMAL CONTROLLER USING PONTRYAGIN'S MINIMUM PRINCIPLE[17]

Let the system dynamics be of the form

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{B.1}$$

Here $\underline{x}$ is an n-vector, $\underline{u}$ is an r-vector, A is an nxn matrix and B is an nxr matrix.

The cost to be minimized is taken as

$$J = \frac{1}{2}\int_0^\infty (\underline{x}^T\,Q\,\underline{x} + \underline{u}^T\,R\,\underline{u})\,dt \tag{B.2}$$

where Q and R are nxn and rxr matrices respectively. We augment the cost J by adding $\lambda^T(A\underline{x} + B\underline{u} - \dot{\underline{x}})$ where $\underline{\lambda}$ is an nx1 costate vector.

$$J' = \int_0^\infty \left[\frac{1}{2}(\underline{x}^T\,Q\,\underline{x} + \underline{u}^T\,R\,\underline{u}) + \underline{\lambda}^T(A\,\underline{x} + B\,\underline{u} - \dot{\underline{x}})\right] dt \tag{B.3}$$

Let the Hamiltonian be defined as

$$H = \frac{1}{2}(\underline{x}^T\,Q\,\underline{x} + \underline{u}^T\,R\,\underline{u}) + \underline{\lambda}^T(A\,\underline{x} + B\,\underline{u}) \tag{B.4}$$

Also let the costate vector $\underline{\lambda}$ satisfy the differential equation

$$\dot{\underline{\lambda}} = -\frac{\partial H}{\partial \underline{x}}(\underline{x}, \underline{\lambda}, \underline{u}) \tag{B.5}$$

Equation (B.1) can also be written as

$$\dot{\underline{x}} = \frac{\partial H}{\partial \lambda}(\underline{x}, \underline{\lambda}, \underline{u}) \tag{B.6}$$

Let U be a set of all admissible control vectors and $\underline{u}^*$ one such admissible control, $\underline{\lambda}^*$ the corresponding solution of (B.5). Then Pontryagin's minimum principle says that for $\underline{u}^*$ to minimize H,

$$H^* = H(\underline{x}^*, \underline{u}^*, \underline{\lambda}^*) = \min_{u \in U} H(\underline{x}^*, \underline{u}, \underline{\lambda}^*) \tag{B.7}$$

where $\underline{x}^*$ is the solution of (B.1) with $\underline{u}^*$ substituted therein, for $\underline{u}$. Also minimization of H in the above manner is required for the minimization of J. A necessary condition for minimizing H as given by (B.4) is

$$\frac{\partial H}{\partial \underline{u}} = 0 \tag{B.8}$$

On performing the differentiation given by (B.8), we get,

$$R\,\underline{u} + B^T \underline{\lambda} = 0 \tag{B.9}$$

or

$$\underline{u}^* = -R^{-1} B^T \underline{\lambda} \tag{B.10}$$

Using the value of $\underline{u}^*$ given in (B.10), (B.5) and (B.6) give the differential equations that $\underline{x}^*$ and $\underline{\lambda}^*$ must satisfy

$$\dot{\underline{x}}^* = A\,\underline{x}^* - B\,R^{-1}\,B^T\,\underline{\lambda}^* \tag{B.11}$$

$$\dot{\underline{\lambda}}^* = -A^T\,\underline{\lambda}^* - Q\,\underline{x}^* \tag{B.12}$$

We would like $\underline{u}$ as a function of $\underline{x}$; so we assume

$$\underline{\lambda} = P\,\underline{x} \tag{B.13}$$

and try to determine the conditions on P. Substitution for the costate in terms of $\underline{x}$ into (B.11) and (B.12) gives

$$\dot{P}\,\underline{x}^* + P\,\dot{\underline{x}}^* = -A^T\,P\,\underline{x}^* - Q\,\underline{x}^* \tag{B.14}$$

$$\underline{\dot{x}}^* = A\,\underline{x}^* - B\,R^{-1}\,B^T\,P\,\underline{x}^* \tag{B.15}$$

Substituting $\underline{\dot{x}}^*$ into (B.14) and factoring out on the right the vector $\underline{x}^*$, gives the nxn matrix differential equation

$$\dot{P} = -P\,A - A^T\,P + P\,B\,R^{-1}\,B^T\,P - Q \tag{B.16}$$

This matrix differential equation is called the matrix Riccati equation. For the infinite time problem, the Riccati equation has a steady state solution for which

$$\dot{P} = 0 \tag{B.17}$$

Once the steady state solution to the matrix Riccati equation is found

$$\underline{u}^* = -R^{-1}\,B^T\,\underline{\lambda} = -R^{-1}\,B^T\,P\,\underline{x} \tag{B.18}$$

and if we let

$$K = R^{-1}\,B^T\,P \tag{B.19}$$

then

$$\underline{u}^* = -K\,\underline{x} \tag{B.20}$$

which is in the desired form.

APPENDIX C

SOLUTION OF MATRIX RICCATI EQUATION BY NEWTON-RAPHSON METHOD (BLACKBURN - MAN ALGORITHM)

The matrix differential equation that one comes across in the application of optimal control theory to dynamical systems is given by

$$\dot{P} = -PA - A^T P + P B R^{-1} B^T P - Q \tag{C.1}$$

Here A is the system open loop matrix, B is the control distribution matrix, Q weights the state variables and R weights the control variables. Both Q and R occur in the performance integral. P is the matrix for which solution is sought for. In the case of an infinite time problem, (C.1) reduces to the matrix quadratic equation given by

$$0 = -P A - A^T P + P B R^{-1} B^T P - Q \tag{C.2}$$

Many methods exist in modern control literature for the solution of this matrix quadratic equation. In one of them Blackburn[11] has applied the Newton-Raphson method for the iterative solution of (C.2). His iterative scheme is essentially as follows:

$$\left[P_{k+1}\right] = \left[P_k\right] + \{ I x (A-GP_k)^T + (A - GP_k)^T x I \} \left[-P_k A - A^T P_k + P_k G P_k - Q\right] \tag{C.3}$$

where $G = B R^{-1} B^T$ and the subscript k indicates the kth iteration, and the notations $\left[C\right]$ and $\{ D \}$ denote, respectively, a $\frac{1}{2} n(n+1)$ dimensional column-vector and a

$\frac{1}{2}n(n+1) \times \frac{1}{2}n(n+1)$ matrix formulated from the $n \times n$ matrix C and the $n^2 \times n^2$ matrix D. Man[12] has further simplified the above method as follows:

Writing (C.3) as

$$\left[P_{k+1}\right] = \left[P_k\right] + \{I \times (A-GP_k)^T + (A-GP_k)^T \times I\}^{-1} \left[-P_k(A-GP_k)-(A-GP_k)^T P_k - P_k G P_k - Q\right] \quad \text{(C.4)}$$

Regrouping the terms in (C.4) gives

$$\left[P_{k+1}\right] = \{I \times (A-GP_k)^T + (A-GP_k)^T \times I\}^{-1} \left[-P_k G P_k - Q\right] + \left[P_k\right] - \{I \times (A-GP_k)^T + (A-GP_k)^T \times I\}^{-1} \left[P_K(A-GP_k)+(A-GP_k)^T P_k\right] \quad \text{.. (C.5)}$$

With the use of the Kronecker product, it can be shown that

$$\left[P_k(A-GP_k)+(A-GP_k)^T P_k\right] = \{I \times (A-GP_k)^T + (A-GP_k)^T \times I\}\left[P_k\right] \quad \text{.. (C.6)}$$

Substituting (C.6) in (C.5) yields

$$\left[P_{k+1}\right] = \{I \times (A-GP_k)^T + (A-GP_k)^T \times I\}^{-1} \left[-P_k G P_k - Q\right] \quad \text{(C.7)}$$

In view of (C.6), (C.7) can be further simplified to give the desired iterative scheme

$$P_{k+1}(A-GP_k)+(A-GP_k)^T P_{k+1} = -P_k G P_k - Q \quad \text{(C.8)}$$

Equation (C.8) is nothing but the matrix Lyapunov equation of the type

$$S F + F^T S = -T \quad \text{(C.9)}$$

for which many methods exist.

One problem still remains viz. finding the initial solution for the iterative scheme given by (C.8). The procedure usually followed is to solve the $\frac{1}{2}n(n+1)$ simultaneous scalar differential equations resulting out of the matrix differential equations given by (C.1) till such time that the resulting matrix P stabilizes[20] the closed-loop matrix $(A-B\,R^{-1}\,B^T\,P)$. The matrix P thus got is used as the initial solution for the equation given by (C.8).

APPENDIX D

FINDING INITIAL SOLUTION FOR MATRIX RICCATI EQUATION

It is well known[20] that the convergence of the iterative process given by (C.8) of Appendix C is assured if the initial solution matrix P_o stabilizes the closed loop system matrix $(A-BR^{-1}B^TP_o)$.

Kleinman[37] has shown that if the original system given by

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{D.1}$$

is controllable, then the matrix W^{-1} will stabilize the matrix A where

$$W = \int_o^T e^{-A\tau}\, B\, B^T\, e^{-A^T\tau}\, d\tau\ , \quad T = \text{arbitrary} \tag{D.2}$$

Thus the appropriate initial solution will now be $P_o = W^{-1}$. Further, in (D.2) the matrix exponential e^{At} can be computed in a closed form by the algorithm given by Vidyasagar[36] which is given below.

Determination of e^{At} in a Closed Form

We can write e^{At} as

$$e^{At} = \sum_{i=o}^{n-1} \alpha_i\, A^i \tag{D.3}$$

where α_i, $i = o, \ldots, n-1$ are functions of t. It is the aim of the algorithm given by Vidyasagar to determine these

scalar functions. The algorithm runs as below:

(i) Find the eigenvalues $\lambda_1, \ldots, \lambda_n$ of the matrix A.

(ii) Compute $\left(\frac{\partial P}{\partial s}\right)_{\lambda_j} = \prod_{i \neq j} (\lambda_j - \lambda_i)$ (D.4)

where $P(s) = \prod_{i=1}^{n} (s - \lambda_i) = \sum_{i=0}^{n} a_i s^i$ (D.5)

(iii) Define

$$C_m(t) = \sum_{j=1}^{n} \lambda_j^m \left\{ \frac{e^{\lambda_j t}}{\left(\frac{\partial P}{\partial s}\right)_{\lambda_j}} \right\}, \quad m = 0, \ldots, n-1 \qquad (D.6)$$

Then the scalar functions α_i are determined by

(iv) $$\alpha_{i-1} = \sum_{k=i}^{n} a_k C_{k-i}(t) \qquad (D.7)$$

$i = 1, \ldots, n$

(v) Finally e^{At} is computed from (D.3).

APPENDIX E

(i) Matrices and Vectors Described in Chapter 4

The elements of the right hand vector $\underline{\eta}$ (20x1) of (4.69) are described below. Let

$$\rho = \frac{\tau_{12}\ f^{*}x0.5x\ \pi^{2}}{4H_1}$$

Then

$\eta\,(1) = -2\rho P_{12}$

$\eta\,(2) = -2\rho\ P_{22}$

$\eta\,(3) = 0.0$

$\eta\,(4) = -2\rho P_{23}$

$\eta\,(5) = 0.0$

$\eta\,(6) = -2\rho P_{24}$

$\eta\,(7) = 0.0$

$\eta\,(8) = 0.0$

$\eta\,(9) = 0.0$

$\eta\,(10) = 0.0$

$\eta\,(11) = 0.0$

$\eta\,(12) = 0.0$

$\eta\,(13) = 0.0$

$\eta\,(14) = 0.0$

$\eta\,(15) = 0.0$

$\eta\,(16) = 0.0$

$\eta\,(17) = 0.0$

$\eta\,(18) = 0.0$

$\eta\,(19) = 0.0$

$\eta\,(20) = 0.0$

Here P_{ij}'s are the elements of the matrix P_* computed as under (4.64).

Now let $D_{ij} = 2a_{ii} + a_{jj}$

$D_{ijk} = a_{ii} + a_{jj} + a_{kk}$

$i = 1,2,3,4$
$j = 1,2,3,4$
$k = 1,2,3,4$

Also let the elements a_{ij} described below to be the elements of the matrix $A_* = (\bar{A} + BK)$ computed as under (4.66). Then the matrix ν (20x20) of (4.69) is described as

$$\left[\begin{array}{cccccccccccccccccccc}
3a_{11} & a_{21} & 0 & a_{31} & 0 & a_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3a_{12} & D_{12} & 2a_{21} & a_{32} & 0 & a_{42} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{31} & a_{41} & 0 & 0 \\
0 & 2a_{12} & D_{21} & 0 & 0 & 0 & 0 & 3a_{21} & a_{31} & 0 & a_{41} & 0 & 0 & 0 & 0 & 0 & a_{32} & a_{42} & 0 & 0 \\
3a_{13} & a_{23} & 0 & D_{13} & 2a_{31} & a_{43} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{21} & 0 & a_{41} & 0 \\
0 & 0 & 0 & 2a_{13} & D_{31} & 0 & 0 & 0 & 0 & a_{21} & 0 & 0 & 3a_{31} & a_{41} & 0 & 0 & a_{23} & 0 & a_{43} & 0 \\
3a_{14} & a_{24} & 0 & a_{34} & 0 & D_{14} & 2a_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{21} & a_{31} & 0 \\
0 & 0 & 0 & 0 & 0 & 2a_{14} & D_{41} & 0 & 0 & 0 & 0 & a_{21} & 0 & 0 & a_{31} & 3a_{41} & 0 & a_{24} & a_{34} & 0 \\
0 & 0 & a_{12} & 0 & 0 & 0 & 0 & 3a_{22} & a_{32} & 0 & a_{42} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{13} & 0 & 0 & 0 & 0 & 3a_{23} & D_{23} & 2a_{32} & a_{43} & 0 & 0 & 0 & 0 & 0 & a_{12} & 0 & 0 & a_{42} \\
0 & 0 & 0 & 0 & a_{12} & 0 & 0 & 0 & 2a_{23} & D_{32} & 0 & 0 & 3a_{32} & a_{42} & 0 & 0 & a_{13} & 0 & 0 & a_{43} \\
0 & 0 & a_{14} & 0 & 0 & 0 & 0 & 3a_{24} & a_{34} & 0 & D_{24} & 2a_{42} & 0 & 0 & 0 & 0 & 0 & a_{12} & 0 & a_{32} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{12} & 0 & 0 & 0 & 2a_{24} & D_{42} & 0 & 0 & a_{32} & 3a_{42} & 0 & a_{14} & 0 & a_{34} \\
0 & 0 & 0 & 0 & a_{13} & 0 & 0 & 0 & 0 & a_{23} & 0 & 0 & 3a_{33} & a_{43} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{14} & 0 & 0 & 0 & 0 & a_{24} & 0 & 0 & 3a_{34} & D_{34} & 2a_{43} & 0 & 0 & 0 & a_{13} & a_{23} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{13} & 0 & 0 & 0 & 0 & a_{23} & 0 & 2a_{34} & D_{43} & 3a_{43} & 0 & 0 & a_{14} & a_{24} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{14} & 0 & 0 & 0 & 0 & a_{24} & 0 & 0 & a_{34} & 3a_{44} & 0 & 0 & 0 & 0 \\
0 & 2a_{13} & 2a_{23} & 2a_{12} & 2a_{32} & 0 & 0 & 0 & 2a_{21} & 2a_{31} & 0 & 0 & 0 & 0 & 0 & 0 & D_{123} & a_{43} & a_{42} & a_{41} \\
0 & 2a_{14} & 2a_{24} & 0 & 0 & 2a_{12} & 2a_{42} & 0 & 0 & 0 & 2a_{21} & 2a_{41} & 0 & 0 & 0 & 0 & a_{34} & D_{124} & a_{32} & a_{31} \\
0 & 0 & 0 & 2a_{14} & 2a_{34} & 2a_{13} & 2a_{43} & 0 & 0 & 0 & 0 & 0 & 0 & 2a_{31} & 2a_{41} & 0 & a_{24} & a_{23} & D_{134} & a_{21} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2a_{24} & 2a_{34} & 2a_{23} & 2a_{43} & 0 & 2a_{32} & 2a_{42} & 0 & a_{14} & a_{13} & a_{12} & D_{234}
\end{array}\right]$$

(ii) Matrices and Vectors Described in Chapter 5

The elements of the matrix $\hat{A}$ (35x35) given in (5.32) are described below row by row. Let $E_{ij} = 2f_{ii} + f_{jj}$

$$E_{ijk} = f_{ii} + f_{jj} + f_{kk} \qquad \begin{array}{l} i = 1,2,3,4,5 \\ j = 1,2,3,4,5 \\ k = 1,2,3,4,5 \end{array}$$

Row 1: $3f_{11}$ f_{21} 0 f_{31} 0 f_{41} 0 f_{51} 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Row 2: $3f_{12}$ E_{12} $2f_{21}$ f_{32} 0 f_{42} 0 f_{52} 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 f_{31} f_{41} f_{51} 0 0 0 0 0 0 0

Row 3: 0 $2f_{12}$ E_{21} 0 0 0 0 0 0 $3f_{21}$ f_{31} 0 f_{41} 0 f_{51} 0 0 0 0
0 0 0 0 0 0 f_{32} f_{42} f_{52} 0 0 0 0 0 0 0

Row 4: $3f_{13}$ f_{23} 0 E_{13} $2f_{31}$ f_{43} 0 f_{53} 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 f_{21} 0 0 f_{41} f_{51} 0 0 0 0 0

Row 5: 0 0 0 $2f_{13}$ E_{31} 0 0 0 0 0 0 f_{21} 0 0 0 0 $3f_{31}$ f_{41} 0
f_{51} 0 0 0 0 0 f_{23} 0 0 f_{43} f_{53} 0 0 0 0 0

Row 6: $3f_{14}$ f_{24} 0 f_{34} 0 E_{14} $2f_{41}$ f_{54} 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 f_{21} 0 f_{31} 0 f_{51} 0 0 0 0

Row 7 0 0 0 0 0 $2f_{14}$ E_{41} 0 0 0 0 0 0 f_{21} 0 0 0 0 f_{31}
0 0 $3f_{41}$ f_{51} 0 0 0 f_{24} 0 f_{34} 0 f_{54} 0 0 0 0

Row 8:	$3f_{15}$	f_{25}	0	f_{35}	0	f_{45}	0	E_{15}	$2f_{51}$	0	0	0	0	0	0	0	0	0	0
				0	0	0	0	0	0	0	0	f_{21}	0	f_{31}	f_{41}	0	0	0	0
Row 9:	0	0	0	0	0	0	0	$2f_{15}$	E_{51}	0	0	0	0	0	0	f_{21}	0	0	0
				0	f_{31}	0	0	f_{41}	$3f_{51}$	0	0	f_{25}	0	f_{35}	f_{45}	0	0	0	0
Row 10:	0	0	f_{12}	0	0	0	0	0	0	$3f_{22}$	f_{32}	0	f_{42}	0	f_{32}	0	0	0	0
				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Row 11:	0	0	f_{13}	0	0	0	0	0	0	$3f_{23}$	E_{23}	$2f_{32}$	f_{43}	0	f_{53}	0	0	0	0
				0	0	0	0	0	0	f_{12}	0	0	0	0	0	f_{42}	f_{52}	0	0
Row 12:	0	0	0	0	f_{12}	0	0	0	0	0	$2f_{23}$	E_{32}	0	0	0	0	$3f_{32}$	f_{42}	0
				f_{52}	0	0	0	0	0	f_{13}	0	0	0	0	0	f_{43}	f_{53}	0	0
Row 13:	0	0	f_{14}	0	0	0	0	0	0	$3f_{24}$	f_{34}	0	E_{24}	$2f_{42}$	f_{54}	0	0	0	0
				0	0	0	0	0	0	0	f_{12}	0	0	0	0	f_{32}	0	f_{52}	0
Row 14:	0	0	0	0	0	0	f_{12}	0	0	0	0	0	$2f_{24}$	E_{42}	0	0	0	0	f_{32}
				0	0	$3f_{42}$	f_{52}	0	0	0	f_{14}	0	0	0	0	f_{34}	0	f_{54}	0
Row 15:	0	0	f_{15}	0	0	0	0	0	0	$3f_{25}$	f_{35}	0	f_{45}	0	E_{25}	$2f_{52}$	0	0	0
				0	0	0	0	0	0	0	0	f_{12}	0	0	0	0	f_{32}	f_{42}	0
Row 16:	0	0	0	0	0	0	0	0	f_{12}	0	0	0	0	0	$2f_{25}$	E_{52}	0	0	0
				0	f_{32}	0	0	f_{42}	$3f_{52}$	0	0	f_{15}	0	0	0	0	f_{35}	f_{45}	0

Row 17: $0 \quad 0 \quad 0 \quad 0 \quad f_{13} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{23} \quad 0 \quad 0 \quad 0 \quad 0 \quad 3f_{33} \quad f_{43} \quad 0$

$f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

Row 18: $0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{24} \quad 0 \quad 0 \quad 0 \quad 0 \quad 3f_{34} \quad E_{34} \quad 2f_{43}$

$f_{54} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{13} \quad 0 \quad 0 \quad f_{23} \quad 0 \quad 0 \quad f_{53}$

Row 19: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{13} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{23} \quad 0 \quad 0 \quad 0 \quad 2f_{34} \quad E_{43}$

$0 \quad 0 \quad 3f_{43} \quad f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad 0 \quad 0 \quad f_{24} \quad 0 \quad 0 \quad f_{54}$

Row 20: $0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{25} \quad 0 \quad 0 \quad 0 \quad 0 \quad 3f_{35} \quad f_{45} \quad 0$

$E_{35} \quad 2f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{13} \quad 0 \quad 0 \quad f_{23} \quad 0 \quad f_{43}$

Row 21: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{13} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{23} \quad 0 \quad 0 \quad 0$

$2f_{35} \quad E_{53} \quad 0 \quad 0 \quad f_{43} \quad 3f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad 0 \quad f_{25} \quad 0 \quad f_{45}$

Row 22: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{24} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{34}$

$0 \quad 0 \quad 3f_{44} \quad f_{54} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

Row 23: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{25} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{35}$

$0 \quad 0 \quad 3f_{45} \quad E_{45} \quad 2f_{54} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad 0 \quad 0 \quad f_{24} \quad f_{34}$

Row 24: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{24} \quad 0 \quad 0 \quad 0$

$0 \quad f_{34} \quad 0 \quad 0 \quad E_{54} \quad 3f_{54} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad 0 \quad f_{25} \quad f_{35}$

Row 25: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{25} \quad 0 \quad 0 \quad 0$

$0 \quad f_{35} \quad 0 \quad 0 \quad f_{45} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

Row 26: $0 \quad 2f_{13} \quad 2f_{23} \quad 0 \quad 2f_{32} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{21} \quad 2f_{31} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad E_{123} \quad f_{43} \quad f_{53} \quad f_{42} \quad f_{52} \quad 0 \quad f_{41} \quad f_{51} \quad 0 \quad 0$

Row 27: $0 \quad 2f_{14} \quad 2f_{24} \quad 0 \quad 0 \quad 2f_{12} \quad 2f_{42} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{21} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{34} \quad E_{124} \quad f_{54} \quad 0 \quad 0 \quad f_{52} \quad f_{31} \quad 0 \quad f_{51} \quad 0$

Row 28: $0 \quad 2f_{15} \quad 2f_{25} \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{12} \quad 2f_{52} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{21} \quad 2f_{51} \quad 0 \quad 0 \quad 0$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{35} \quad f_{45} \quad E_{125} \quad 0 \quad f_{32} \quad f_{42} \quad 0 \quad f_{31} \quad f_{41} \quad 0$

Row 29: $0 \quad 0 \quad 0 \quad 2f_{14} \quad 2f_{34} \quad 2f_{13} \quad 2f_{43} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{31} \quad 2f_{41}$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{24} \quad f_{23} \quad 0 \quad E_{134} \quad f_{54} \quad f_{53} \quad f_{21} \quad 0 \quad 0 \quad f_{51}$

Row 30: $0 \quad 0 \quad 0 \quad 2f_{15} \quad 2f_{35} \quad 0 \quad 0 \quad 2f_{13} \quad 2f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$2f_{31} \quad 2f_{51} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{25} \quad 0 \quad f_{23} \quad f_{45} \quad E_{135} \quad f_{43} \quad 0 \quad f_{21} \quad 0 \quad f_{41}$

Row 31 $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{15} \quad 2f_{45} \quad 2f_{14} \quad 2f_{54} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$0 \quad 0 \quad 0 \quad 2f_{41} \quad 2f_{51} \quad 0 \quad 0 \quad f_{25} \quad f_{24} \quad f_{35} \quad f_{34} \quad E_{145} \quad 0 \quad 0 \quad f_{21} \quad f_{31}$

Row 32 $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{24} \quad 2f_{34} \quad 2f_{23} \quad 2f_{13} \quad 0 \quad 0 \quad 0 \quad 2f_{32} \quad 2f_{42}$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{14} \quad f_{13} \quad 0 \quad f_{12} \quad 0 \quad 0 \quad E_{234} \quad f_{54} \quad f_{53} \quad f_{52}$

Row 33 $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{25} \quad 2f_{35} \quad 0 \quad 0 \quad 2f_{23} \quad 2f_{53} \quad 0 \quad 0 \quad 0$

$2f_{32} \quad 2f_{52} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad 0 \quad f_{13} \quad 0 \quad f_{12} \quad 0 \quad f_{45} \quad E_{235} \quad f_{43} \quad f_{42}$

Row 34: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{25} \quad 2f_{45} \quad 2f_{24} \quad 2f_{54} \quad 0 \quad 0 \quad 0$

$0 \quad 0 \quad 0 \quad 2f_{42} \quad 2f_{52} \quad 0 \quad 0 \quad f_{15} \quad f_{14} \quad 0 \quad 0 \quad f_{12} \quad f_{35} \quad f_{34} \quad E_{245} \quad f_{32}$

Row 35: $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2f_{35} \quad 2f_{45}$

$2f_{34} \quad 2f_{54} \quad 0 \quad 2f_{43} \quad 2f_{53} \quad 0 \quad 0 \quad 0 \quad 0 \quad f_{15} \quad f_{14} \quad f_{13} \quad f_{25} \quad f_{24} \quad f_{23} \quad E_{345}$

The elements of the right hand vector $\hat{\varepsilon}$ (35x1) are computed as follows: Let

$$t_x = \frac{0.5f^* \quad \tau_{12} \quad \pi^2}{H_1} \quad \text{and} \quad t_y = -\frac{0.866f^* \; \tau_{12} \, \pi}{H_1}$$

Let

$q_a = t_x(c_{12} - c_{16})$

$q_b = t_y(c_{12} - c_{16})$

$q_c = t_x(c_{22} - c_{26})$

$q_d = t_y(c_{22} - c_{26})$

$q_e = t_x(c_{32} - c_{36})$

$q_f = t_y(c_{32} - c_{36})$

$q_g = t_x(c_{42} - c_{46})$

$q_h = t_y(c_{42} - c_{46})$

$q_i = t_x(c_{52} - c_{56})$

$q_j = t_y(c_{52} - c_{56})$

Here c_{ij}'s are the elements of the transformation matrix C (5x8). Also let

$f_a = p_{11} \; q_a + p_{12} \; q_c + p_{13} \; q_e + p_{14} \; q_g + p_{15} \; q_i$

$f_b = p_{12} \; q_a + p_{22} \; q_c + p_{23} \; q_e + p_{24} \; q_g + p_{25} \; q_i$

$f_c = p_{13} \; q_a + p_{23} \; q_c + p_{33} \; q_e + p_{34} \; q_g + q_{35} \; q_i$

$f_d = p_{14} \; q_a + p_{24} \; q_c + p_{34} \; q_e + p_{44} \; q_g + p_{45} \; q_i$

$f_e = p_{15} \; q_a + p_{25} \; q_c + p_{35} \; q_e + p_{45} \; q_g + p_{55} \; q_i$

where p_{ij}'s are the elements of the solution B_* of the matrix Riccati equation. Also let $\hat{H} = C^T(CC^T)^{-1}$ and h_{ij}'s be its elements. Let

$w_1 = h_{11} - h_{51}$

$w_2 = h_{12} - h_{52}$

$w_3 = h_{13} - h_{53}$

$w_4 = h_{14} - h_{54}$

$w_5 = h_{15} - h_{55}$

Then $\hat{\varepsilon}(1) = 2w_1^2 f_a$

$\hat{\varepsilon}(2) = 4w_1w_2f_a + 2w_1^2f_b$

$\hat{\varepsilon}(3) = 2w_2^2f_a + 4w_1w_2f_b$

$\hat{\varepsilon}(4) = 4w_1w_3f_a + 2w_1^2f_c$

$\hat{\varepsilon}(5) = 2w_3^2f_a + 4w_1w_3f_c$

$\hat{\varepsilon}(6) = 4w_1w_4f_a + 2w_1^2f_d$

$\hat{\varepsilon}(7) = 2w_4^2 f_a + 4w_1w_4f_d$

$\hat{\varepsilon}(8) = 4w_1w_5f_a+2w_1^2f_e$

$\hat{\varepsilon}(9) = 2w_5^2 f_a+4w_1w_5f_e$

$\hat{\varepsilon}(10) = 2w_2^2f_b$

$\hat{\varepsilon}(11) = 4w_2w_3f_b+2w_2^2f_c$

$\hat{\varepsilon}(12) = 2w_3^2f_b + 4w_2w_3f_c$

$\hat{\varepsilon}(13) = 4w_2w_4f_b+2w_2^2 f_d$

$\hat{\varepsilon}(14) = 2w_4^2f_b+4w_2w_4f_d$

$\hat{\varepsilon}(15) = 4w_2w_5f_b+2w_2^2f_e$

$\hat{\varepsilon}(16) = 2w_5^2f_b+4w_2w_5f_e$

$\hat{\varepsilon}(17) = 2w_3^2f_c$

$\hat{\varepsilon}(18) = 4w_3w_4f_c+2w_3^2f_d$

$\hat{\varepsilon}(19) = 2w_4^2 f_c + 4w_3w_4f_d$

$\hat{\varepsilon}(20) = 4w_3w_5f_c+2w_3^2f_e$

$\hat{\varepsilon}(21) = 2w_5^2f_c+4w_3w_5f_e$

$\hat{\varepsilon}(22) = 2w_4^2 f_d$

$\hat{\varepsilon}(23) = 4w_4w_5f_d+ 2w_4^2f_e$

$\hat{\varepsilon}(24) = 2w_5^2f_d + 4w_4w_5f_e$

$\hat{\varepsilon}(25) = 2w_5^2f_e$

$\hat{\varepsilon}(26) = 4w_2w_3f_a+4w_1w_3f_b+4w_1w_2f_c$

$\hat{\varepsilon}(27) = 4w_2w_4f_a + 4w_1w_4f_b+4w_1w_2f_d$

$\hat{\varepsilon}(28) = 4w_2w_5f_a + 4w_1w_5f_b + 4w_1w_2f_e$

$\hat{\varepsilon}(29) = 4w_3w_4f_a+ 4w_1w_4 f_c + 4w_1w_3f_d$

$\hat{\varepsilon}(30) = 4w_3w_5f_a + 4w_1w_5f_c + 4w_1w_3f_e$

$\hat{\varepsilon}(31) = 4w_4w_5f_a + 4w_1w_5f_d + 4w_1w_4f_e$

$\hat{\varepsilon}(32) = 4w_3w_4f_b + 4w_2w_4f_c + 4w_2w_3f_d$

$$\hat{\varepsilon}(33) = 4w_3w_5f_b + 4w_2w_5f_c + 4w_2w_3\ f_e$$

$$\hat{\varepsilon}(34) = 4w_4w_5f_b + 4w_2w_5f_d + 4w_2w_4f_e$$

$$\hat{\varepsilon}(35) = 4w_4w_5f_c + 4w_3w_5f_d + 4w_3w_4f_e$$

The elements of the right hand vector are computed as

$$\hat{\varepsilon}(10) = 0.26673\text{x}10^1$$

$$\hat{\varepsilon}(11) = 0.33333\text{x}10^1$$

$$\hat{\varepsilon}(12) = -0.55140\text{x}10^1$$

$$\hat{\varepsilon}(17) = -0.69565\text{x}10^1$$

The rest of the elements being zero. Also the elements of the solution vector $\underline{m}$ are computed as

$$m(10) = -0.44360\text{x}10^0$$

$$m(11) = 0.56631\text{x}10^{-1}$$

$$m(12) = -0.12688\text{x}10^1$$

$$m(17) = 0.96424\text{x}10^0$$

The rest of the elements being zero.

CURRICULUM VITAE

1. Candidate's name: Vemparala Ramachandra Moorthi

2. Academic Background:

Degree	Specialization	Institution/University	Year
B.Sc.	Physics	Andhra University	1950
B.E.	Electrical Engineering	Osmania University	1953
M.Tech.	Power System Engineering	Indian Institute of Technology, Kharagpur	1967

3. Publications:

(i) "Optimum Megawatt Frequency Control of a Three Area System" (With M. Ramamoorty), Presented at the IEEE Summer Power Meeting, July 1972 (C72 490-1).

(ii) "Suboptimal and Near Optimal Control of a Load Frequency Control System" (with R.P.Aggarwal), Accepted for Publication in Proc. IEE (London).

(iii) "Suboptimal Regulation of Nonlinear Load Frequency Control Systems" (with R.P.Aggarwal). Accepted for Presentation as Conference Paper at the IEEE Winter Power Meeting, Jan.-Feb. 1973 (C73 099-9).